

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Find the value of  $\lim_{h \rightarrow 0} \frac{\cos(\pi + h) + 1}{h}$ .

Answer : \_\_\_\_\_

2. Find the area of the region bounded by the curves  $y = \sin x$ ,  $y = \cos x$ ,  $x = 0$ , and  $x = \pi/2$ .

Answer : \_\_\_\_\_

3. Find the maximum rate of change of  $f(x, y) = \sin(xy)$  at  $(0, 1)$ .

Answer : \_\_\_\_\_

4. Let  $g(x)$  be the inverse function of  $f(x) = 3 + x^2 + \tan(\pi x/2)$ ,  $-1 < x < 1$ . Find  $g'(3)$ .

Answer : \_\_\_\_\_

5. Let  $f(x, y) = \begin{cases} \frac{x^2+2y^3}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$  Find  $f_y(0, 0)$ .

Answer : \_\_\_\_\_

6. Find the interval of convergence of the series  $\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$ .

Answer : \_\_\_\_\_

7. Convert the integral  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} dz dy dx$  to an equivalent integral in spherical coordinates.

Answer : \_\_\_\_\_ (Do not evaluate the integral).

8. Evaluate the iterated integral  $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dx dy$ .

Answer : \_\_\_\_\_

參考用

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Find the limit  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\sqrt{n^2 - k^2}}{n^2}$ .

2. Evaluate the line integral  $\oint_C y^3 dx - x^3 dy$  where  $C$  is the circle  $x^2 + y^2 = 4$ .

3. Find the extreme values of  $f(x, y) = 2x^2 + 3y^2 - 4x - 5$  on the disk  $x^2 + y^2 \leq 16$ .