所別：天文研究所碩士班 不分組(一般生)　科目：普通物理　共 2 頁　第 1 頁
天文研究所碩士班 不分組(在職生)

本科考試禁用計算器　　*請在試卷答案卷（卡）內作答

參考用

1. (20%) A binary is a system consisting of two stars orbiting around their common center of mass under the influence of their mutual gravitational force. Suppose the two stars move in circular orbits and the separation of them are much larger than the sizes of the stars so they can be considered as particles. Prove that the Kepler's 3rd law for this binary system can be written as

$$\frac{a^3}{P^2}=\frac{G(m_1+m_2)}{4\pi^2}$$

where a is the separation of two stars, P is the orbital period, m_1 and m_2 are the masses of two stars and G is gravitational constant.
Suppose only the orbital period P and the speed of m_1, K_1, are measurable. Show that

$$\frac{K_1^3 P}{2\pi G}=\frac{m_2^3}{(m_1+m_2)^2}$$

and prove this value can be the estimation of the lower limit of m_2

2. (20%) The rotation-powered pulsar is a fast rotating neutron star with a magnetic dipole whose direction is misaligned to the rotational axis. Since the direction of dipole changes with the time, the radiation power is

$$\left.\frac{dE}{dt}\right|_{dipole}=\frac{2M^2\sin^2\alpha}{3c^3}\Omega^4$$

where M is the magnitude of magnetic dipole ($M=|\vec{M}|$), Ω is the spin angular velocity of neutron star, α is angle between magnetic moment and rotational axis, and c is speed of light. However, the only energy resource to support this radiation is to consume its own rotational energy. Consider the neutron star as a rigid body of moment of inertia I. Prove that

$$\dot{\Omega}=\frac{d\Omega}{dt}=-k\Omega^3$$

where k is a positive constant. Furthermore, we also know that the surface magnetic field strength (B) is proportional to M. Show that

$$B\propto\sqrt{P\dot{P}}$$

where P is the spin period of the neutron star and $\dot{P}=dP/dt$ is its time derivative.

注意：背面有試題

3. (15%) A beam of photons whose frequency is ν normally incidents on a mirror which is moving in the opposite direction of the incident photons with constant velocity v shown as Fig. 1. Find the frequency of reflected photons. (Hint: consider the special relativity)

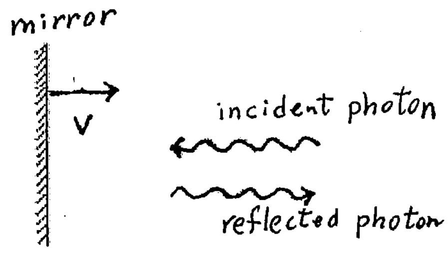

Fig. 1

4. (15%) Two bodies have identical mass m and the specific heat per unit mass $c_v = \frac{1}{m}\left(\frac{dQ}{dT}\right)_v$ but have different absolute temperature T_1 and T_2 ($T_1 > T_2$). Now we want to extract mechanical work from these two bodies by connecting a heat engine. If the volumes of these two bodies are unchanged, what is the maximum work can be extracted from this system and the final temperature for these two bodies?

5. (15%) A circuit composed by two capacitors with capacitances C_1 and C_2, a resister of resistance R and a switch S is shown as Fig. 2. Initially, the capacitor C_1 has charge Q_0 on it but no charge on C_2. At $t = 0$, the switch is closed so the C_2 starts being charged.

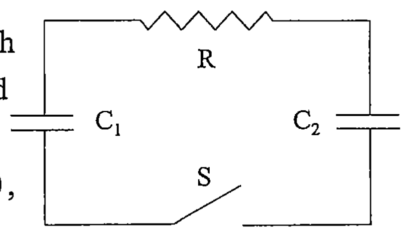

Fig. 2

(a) Find the current I passing through the resister as a function of time. (5%)

(b) Find the final charges on C_1 and C_2 after the charging is complete. (5%)

(c) What are the energies restored at these two capacitors after the charging is complete? How much energy is dissipated by the resistor during this process? (5%)

6. (15%) As shown in Fig. 3, a uniform magnetic field $\vec{B} = B\hat{z}$ is confined to a slab region of thickness t ($0 < x < t, -\infty < y < \infty$). A charged particle of charge q and mass m incidents into this region vertically with velocity $\vec{v} = v\hat{x}$. What is the minimum speed of this particle to make it passing through this magnetic field region? What happen for the particle if its speed less this minimum value?

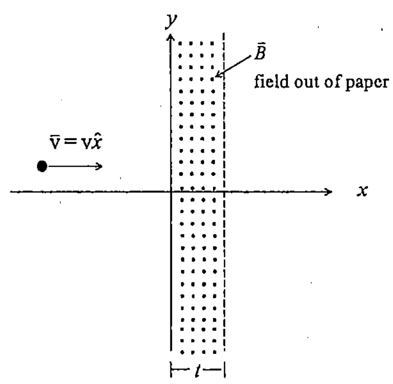

Fig. 3

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