

1. (25 Points)

Let p, q, r be the following sentences: (10 points)

p : "John is at the office."

q : "Joan is at the office."

r : "Laura is at the office."

Use logical connectives to express the following three sentences (represent the given statement symbolically):

(A) "John is not at the office" (7 points)

(B) "If Joan and Laura are at the office then John is at the office." (8 points)

(C) "If John is at the office the either Joan or Laura is at the office." (10 points)

2. (25 Points)

If \sim denote an equivalence relation on a set A , the equivalence class of an element $a \in A$ is the set $\bar{a} = \{x \in A \mid x \sim a\}$.

Let $A = \{0, 2, 4, 8, 16, 32\}$. For $a, b \in A$, define $a \sim b$ if and only if $a*b$ is a perfect square (that is, the square of an integer).

(a) What are the ordered pairs in this relation? (13 points)

(b) For each $a \in A$, find $\bar{a} = \{x \in A \mid x \sim a\}$ (12 points)

3. (25 Points)

(A) Find a necessary and sufficient condition on a natural numbers m and n in order for $K_{m,n}$ to be Eulerian. Prove your answer. (12 points)

(B) Find a necessary and sufficient condition on a natural numbers m and n in order for $K_{m,n}$ to have an Eulerian trail. Assume $m \leq n$. Prove your answer. (13 points)

4. (25 Points)

A sequence is defined recursively by $a_0 = 2, a_1 = 3$ and $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$.

(a) Find the first five terms of this sequence. (5 points)

(b) Guess a formula for a_n . (7 points)

(c) Verify that your guess in (b) is correct. (5 points)

(d) Find a formula for a_n which involves only one preceding term. (8 points)