所別：電機工程學系碩士班 固態組(一般生) 科目：近代物理 共 1 頁 第 1 頁
本科考試禁用計算器 *請在試卷答案卷（卡）內作答

參考用

(1) For the Fermi distribution function $f(\varepsilon)$, show that $\int_0^{E_F} f(\varepsilon)d\varepsilon = E_F$ at zero temperature, where $E_F$ is the Fermi energy. (10%)

(2) Use the Boson distribution function of $n(\varepsilon)$ to drive Stefan law. That is, show that $\alpha\int_0^{\infty} n(\varepsilon)\varepsilon^3 d\varepsilon \propto T^4$, where $\alpha = \frac{2\pi}{c^2 h^3}$ is a temperature ($T$)-independent function. (10%)

(3) Based on the Bragg condition, illustrate the mechanism of band gap in a one dimensional lattice of periodicity $a$. (15%)

(4) In an intrinsic semiconductor, show that the Fermi energy lies at the center of the band gap $E_g$ that separates a valence band and a conduction band, if we assume that the density of holes in the valence band equals to the density of electrons in the conduction band. (15%)

(5) For one dimensional Schroedinger equation with time-independent potential $V(x)$, $\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = \frac{i\hbar\partial\psi(x,t)}{\partial t}$, prove that $\psi(x,t) = e^{-iEt/\hbar}\varphi(x)$ is a solution of wave equation. (10%)

(6) Calculate the eigenvalues and eigenvectors of the Pauli spin matrix $S = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$.(10%) Prove that these eigenvectors are orthogonal. (10%)

(7) Solve the ground state energy for a particle with mass $m$ in an infinite square well under a uniform electric field $F$ by the perturbation method. (20%) The system eigenvalues are $E_n^0 = \frac{\pi^2\hbar^2 n^2}{2ma^2}$ in the absence of $F$. Here $a$ denotes the well width and $n$=1,2,3,4,5