國立中央大學102學年度碩士班考試入學試題卷

所別：電機工程學系碩士班 系統與生醫組(一般生)　科目：信號與系統　共 2 頁 第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

參考用

一、**(15%)** For a continuous-time system with input $x(t)$ and output $y(t)$, the input-output relationship for this system is $y(t) = x(\sin(t))$. Check the following two properties.

(一) (5%) Is this system causal?

(二) (10%) Is this system linear?

二、**(10%)** Suppose that the signal $x(t) = u(t+0.5) - u(t-0.5)$ is convolved with the signal $h(t) = e^{j\omega_0 t}$.

(一) (5%) Determine a value of ω_0 which ensures that $y(0) = 0$, where $y(t) = x(t) * h(t)$.

(二) (5%) Is your answer to part (a) unique?

三、**(10%)** For a linear time-invariant system S and a signal $x(t) = 2e^{-3t}u(t-1)$, determine the impulse response $h(t)$ of S, where $x(t) \to y(t)$ and $\frac{dx(t)}{dt} \to -3y(t) + e^{-2t}u(t)$.

四、**(15 %)** Consider a causal LTI system S whose input $x(t)$ and output $y(t)$ are related by the differential equation

$$a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t) + b_1\frac{dx(t)}{dt} + b_2\frac{d^2x(t)}{dt^2}$$

Show that

$$y(t) = A\int_{-\infty}^{t} y(\tau)d\tau + B\int_{-\infty}^{t}\left(\int_{-\infty}^{\tau} y(\sigma)d\sigma\right)d\tau + Cx(t) + D\int_{-\infty}^{t} x(\tau)d\tau + E\int_{-\infty}^{t}\left(\int_{-\infty}^{\tau} x(\sigma)d\sigma\right)d\tau$$

and express the constants A, B, C, D, and E in terms of the constants a_0, a_1, a_2, b_0, b_1, and b_2.

五、 **(15 %)** Please calculate the discrete-time Fourier Transform $X(e^{j\Omega})$ of signal $x[n]$. The impulse sequence of $x[n]$ is shown in **Fig. 1**, defined as:

$$x[n] = \begin{cases} 1, & \text{for } -5 \le n \le 5 \\ 1/3, & \text{for } -10 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}$$

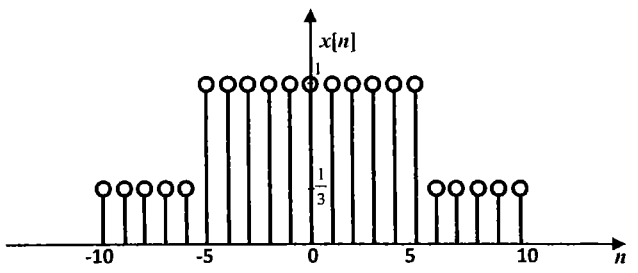

Fig. 1

注意：背面有試題

參考用

六、(15 %) Please prove the following discrete-time Fourier transform (DTFT) properties:

(一) (5%) $x[n]*y[n] \xrightarrow{DTFT} X(e^{j\Omega})\cdot Y(e^{j\Omega})$.

(二) (5%) $nx[n] \xrightarrow{DTFT} j\frac{dX(e^{j\Omega})}{d\Omega}$.

(三) (5%) $\sum_{n=-\infty}^{\infty}|x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|X(e^{j\Omega})\right|^2 d\Omega$.

※Remark: n is the discrete-time index, Ω is the radial frequency, * is discrete-time convolution operator, and $X(e^{j\Omega})$ and $Y(e^{j\Omega})$ are the discrete-time Fourier transforms of discrete-time sequences $x[n]$ and $y[n]$, respectively.

七、(20%) Consider a signal flow graph shown in **Fig. 2**.

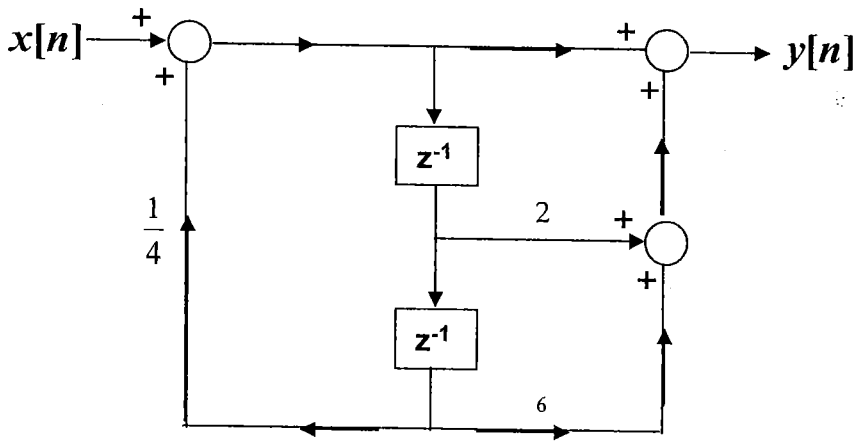

Fig. 2.

(一) (7%) Please find the transfer function H(z).

(二) (10%) If the system is causal, please find the impulse response $h[n]$ of the system.

(三) (3%) Is the system stable? Please explain it.

注意:背面有試題