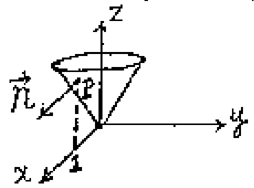
國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 電機工程研究所 甲乙丙丁組 科目: 工程數學 共 2 頁 第 1 頁

1. (10%) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

2.
(5%) (a) Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point P: (2, 1, 3) in the direction of the vector $\vec{a} = \vec{i} - 2\vec{k}$.
(5%) (b) Find a unit vector $\vec{n}$ of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point P: (1, 0, 2).



3.
(5%) (a) Show that the form under the integral sign is exact and evaluate

$$\int_{(0,0,0)}^{(2,2,2)} (2x\,dx + 3y^2 z\,dy + y^3 dz).$$

(5%) (b)Show that a representation $z = f(x, y)$ can be written

$$\vec{r}(u, v) = u\vec{i} + v\vec{j} + f(u, v)\vec{k},$$

and a normal vector of the surface can be written

$$N = -\frac{\partial f}{\partial u}\vec{i} - \frac{\partial f}{\partial v}\vec{j} + \vec{k}.$$

(4) (12%, 3% each) A complex function $f(z)$ is called *entire* if $f(z)$ is analytic for all z in complex plane. Determine whether each of the following complex functions is entire?

(a) $f(z) = \mathrm{Re}\{z^2\}$ (b) $f(z) = z - \bar{z}$ (c) $f(z) = z + \frac{1}{z}$ (d) $f(z) = e^z$

(5) (12%, 6% each) Integrate the following complex functions counterclockwise around the circle $|z| = 2$.

(a) $\frac{z^2}{(z-i)^2}$ (b) $\frac{e^{z^2}}{(z-1)^2}$

(6) (11%,) Consider a random variable X with the probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\{-\frac{x^2}{2}\}$$

for $-\infty < x < \infty$. Define a new random variable Y as follows: When $X = x_0$, Y has the conditional distribution

$$P\{Y = x_0 | X = x_0\} = P\{Y = -x_0 | X = x_0\} = \frac{1}{2}.$$

Find the density function of Y.

(7). (a) Solve the differential equation $yy''=(y')^2$, where both y and y' are positive. (5%).

(b). A differential equation is as $x^2y''+3xy'+y=0$. Is it a linear and homogenous differential equation of y (2%)? Usually, what is the equation's name called by us (3%)? Please find its general solution. (5%).

(8). (a) Using the **convolution formula** to find the inverse of the following Laplace form $H(s)=\dfrac{1}{s^2(s^2+1)}$. (5%).

(b). Find the Laplace transform of $g(t)=\sin(wt+v)$, where w and v are constants. (5%).

(9). Find the Fourier transforms of e^{-ax^2} and xe^{-x^2} respectively, where $a>0$. (5% each).