系所別：電機工程學系　甲組　科目：工程數學
乙、丙組

1. (10%) Solve the initial value problem

$$2\sin(y^2)+xy\cos(y^2)y'=0,\quad y(2)=\sqrt{\pi/2}.$$

2. (10%) Solve the equation
$x^2y''-4xy'+6y=7x^4\sin x$.

3. (8%) Find the Fourier cosine integral of

$$f(x)=e^{-kx},\quad (x>0,k>0).$$

4. (7%) Solve the integral equation

$$y(t)=\sin 2t+\int_0^t y(\tau)\sin 2(t-\tau)d\tau.$$

5. (15%) Evaluate the following integral. Detailed evaluation procedure is required.

$$\int_0^\infty \frac{\cos(2.5x)}{(x^2+1)^2}dx$$

6. Given the function $\frac{(z+1)}{2(z-1)}$, represent it by

(a) (10%) its Maclaurin series, and give the region of validity for the representation;

(b) (5%) its Laurent series for the domain $1<|z|<\infty$.

7. The center of gravity of an object is located at the origin (0, 0, 0) of a Cartesian coordinate system in space. The center of gravity of this object is subject to a force $\vec{F}_1$ in the [5, 2, –4] direction and another force $\vec{F}_2$ in the [8, –6, 5] direction. The magnitude of $\vec{F}_1$ is 6 Newton and that of $\vec{F}_2$ is 10 Newton.
(a) (5 %) What is the net force magnitude acting on the object?
(b) (5 %) In what direction is the net force?

8. (a) (5 %) Let matrix $A=\begin{bmatrix}-3+\sqrt{2} & 5+2\sqrt{2}\\ 2 & 1-\sqrt{2}\end{bmatrix}$. Find the eigenvalues of A.

(b) (5 %) Find a matrix B that is different from A but has the same eigenvalues as A has.
(c) (5 %) Find a matrix C that is different from A and B but has the same eigenvalues as A has. In addition, C must only contain integer elements.

9. Let $\sigma(x,y,z)$ and $V(x,y,z)$ be the electrical conductivity and the electrical potential distribution, respectively, in an object. The unit of σ is $\frac{1}{\Omega\cdot cm}$ and that of V is volt.
(a) (5 %) What is the unit of $\sigma\nabla V$? What is the physical meaning of $\sigma\nabla V$?
(b) (5%) What is the unit of $\nabla\bullet\sigma\nabla V$? Laplace's Equation states that $\nabla\bullet\sigma\nabla V=0$. What is the physical meaning of Laplace's Equation.

(Note: $\nabla=\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k}$ is the gradient operator and $\bullet$ is the notation of the inner product of two vectors.)