所別：電機工程學系碩士班 甲、乙組 丙組 科目：工程數學

1. (10 %) Find the general solution of the differential equation
$y'\cos(y) + x\sin(y) = 2x$.

2. (10 %) Find the general solution of the differential equation
$xy''' + 3y'' = e^x$.

3. (10%) Solve the integral equation
$$y(t) = t + \int_0^t [\sin(t)\cos(\tau)y(\tau) - \cos(t)\sin(\tau)y(\tau)]d\tau .$$

4. (5%) Find the Fourier series of the function
$f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

5. (10%) Prove that the eigenvalues of a Hermitian operator M are real; the eigenvectors of M corresponding to different eigenvalues are orthogonal.

6. (15%) Find the eigenvalues and eigenvectors of matrix
$$M = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}.$$

7. (10%) Find $\exp^{-i\pi A}$, where matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

8. (A) (5%) Show that w = sin z is analytic for every z . Where z is complex variable.
(B) (5%) Give the conformal mapping of problem (A) for the vertical line x =5 in z-plane.
(C) (5%) Repeat problem (B) for the Horizontal line y=5 in z-plane.

9. (A) (5%) Evaluate the following integral around the contour z , the ellipse
$x^2 + 9y^2 = 9$
counterclockwise by Residue Theorm. $\int \frac{z-23}{z^2-4z-5}dz$.
(B) Integrate the following function counterclockwise around the unit circle
(a) (5%) $w = \frac{1}{z}$. (b) (5%) $w = \frac{1}{z^2}$.