國立中央大學97學年度碩士班考試入學試題卷

所別：電機工程學系碩士班 電子組
　　　電機工程學系碩士班 固態組
　　　電機工程學系碩士班 系統與生醫組

科目：工程數學　共　1　頁 第　1　頁

＊請在試卷答案卷（卡）內作答

1. (15%) Evaluate the following integral. Detailed evaluation procedure is required.

$$\int_0^\infty \frac{x\sin(2x)}{x^2+3}\,dx$$

2. (15%) Given the function $\dfrac{(3z+1)}{(z-1)}$, represent it by its Maclaurin series, and give the region of validity for the representation.

3. (15%) Consider the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$. Diagonalize A by similarity transformation. Transition matrices should be given.

4. (a) If $\vec{r}$ and $\vec{n}$ are the position vector and unit normal vector to a closed surface S of a region T, which can be non-smooth, evaluate the surface integral of $\oiint_S \dfrac{\vec{r}}{r^3}\cdot\vec{n}\,dS$, when the coordinate of origin is (i) (5%) outside of S and

   (ii) (5%) inside of S.

   (b) (5%) Consider a charge q at the origin. Its corresponding electric field is

   $$\vec{E} = \frac{q\vec{r}}{4\pi\varepsilon r^3}$$ where ε is a dielectric constant. Based on the results in (a),

   derive Poisson's equation $\nabla^2\phi = \dfrac{Q}{\varepsilon}$ where Q is charge density and ϕ is

   electrostatic potential such that $q = \iiint_T Q\,dV$, (V: volume) and $\vec{E} = \nabla\phi$

   respectively.

5. (15%) Solve the following differential equation

   $$y''' - 6y'' + 12y' - 8y = \sqrt{x}e^{2x}.$$

6. (15%) Find the Laplace transform of the full-wave rectification of $\sin\omega t$ (Show the details of your work).

7. (10%) Using the Fourier integral representation, show that

   $$\int_0^\infty \frac{\cos(\pi\omega/2)\cos x\omega}{1-\omega^2}\,d\omega = \begin{cases} \dfrac{\pi}{2}\cos x & when \quad |x| < \dfrac{\pi}{2} \\ 0 & when \quad |x| > \dfrac{\pi}{2} \end{cases}.$$