1. (15%) Solve the following differential equation

$$y''' - 6y'' + 12y' - 8y = \sqrt{x}e^{2x}.$$

2. (15%) Find the Laplace transform of the full-wave rectification of $\sin \omega t$ (Show the details of your work).

3. (15%) If the matrix A is $\begin{bmatrix} 25 & 40 \\ -12 & -19 \end{bmatrix}$, show that $A^n \begin{bmatrix} 3 \\ -2 \end{bmatrix} = 5^n \begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

   where n is a positive integer.

4. (15%) Calculate $\displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(10 x_1^2 + 2 x_2^2 + 6 x_1 x_2)} dx_1 dx_2$.

5. (15%) Find $\dfrac{d^6(e^{2t}\cos(2t))}{dt^6}$.

6. (15%) Evaluate the integral $\displaystyle\oint_C \left(\frac{ze^{\pi z}}{z^4 - 81} + z^2 e^{\pi/z} \right) dz$, where C is

   the ellipse $16x^2 + y^2 = 16$ and oriented counterclockwise.

7. (10%) Find the Fourier series of the periodic function $f(x) = f(x + 2\pi)$

$$f(x) = \begin{cases} \sin x & if \quad 0 \le x \le \pi \\ -\sin x & if \quad -\pi \le x \le 0 \end{cases}.$$

參考用