科目：資料結構(5002)　　校系所組：中大電機工程學系(電子組)
交大電子研究所(乙組)

1. [12 points] The asymptotic notations, big-oh (O) and big theta (Θ), represent a set of functions but very often we abuse the "+" operator. What are the notions that best describe the following questions? Justify your answers.
   a. [3 pts] $\Theta(n\log n)+O(n^2)$
   b. [3 pts] $\Theta(n!)+O(2^n)$
   c. [3 pts] $\Theta(4n)+O(n^{\log n})+O(2^n)$
   d. [3 pts] $\Theta(\log(n!))+O(n^{1.5})$

2. [12 points] You are asked to implement a queue by stacks. You can only use *stack* operations as follows: *s.push(x)*, *s.pop()* and *s.empty()*, where *s* is a stack instance and *x* is an instance of the element type.
   a. [8 pts] Describe your algorithm in pseudocode for *queue* operations, i.e., *q.insert(x)*, *q.delete()* and *q.empty()* where *q* is a queue instance.
   b. [2 pts] What is the worst-case running time of inserting an element to a queue in (a)? Justify your answer.
   c. [2 pts] What is the worst-case running time of deleting an element from a queue in (a)? Justify your answer.

3. **[12 points, each wrong answer costs you 3 points until 0 point is reached]**
   Let G be a connected and undirected graph with a breadth-first search (BFS) or depth-first search (DFS) tree T, rooted at the vertex s. Let (u, v) be an edge in G but not in T. Label each statement about u and v with *always* (A), *never* (N), or *sometimes* (S), based on whether T is a BFS tree or a DFS tree.

   (Note: $d(w)$ is the number of edges on the path from s to w in the tree)

Relationship of u and v in T	**T is a BFS tree**	**T is a DFS tree**
U and v are ancestor descendant		
$D(u) = d(v)$		
$D(u)$ and $d(v)$ differ by one		
$D(u)$ and $d(v)$ differ by two or more		

參考用

4. [12 points] The abstract data types (ADTs) of a binary search tree, *BST* and *BstNode*, are provided as follows.

```
ADT BST {
    BstNode* root;
}
```

```
ADT BstNode {
    int data;
    BstNode* LeftChild;
    BstNode* RightChild;
}
```

   a. [2 pts] Modify the provided data structure so that it supports to search the k-th largest element.
   b. [5 pts] Describe your search algorithm in pseudocode.
   c. [5 pts] Describe your insertion algorithm in pseudocode.

5. [5 points] Given a binary search tree, suppose the search for key k ends at a leaf. Partition the tree into the three sets: A is the set of keys to the left of the search path, B is the set of keys on the search path, and C is the set of keys to the right of the search path. We claim that for all $a \in A$, $b \in B$ and $c \in C$, it follows that $a \le b \le c$. State whether the claim is true or false. If true, justify your answer. If false, give a counter-example.

注意：背面有試題

參考用

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6. [14 points]

a. [3 pts] Given an undirected graph G with n vertices and m edges, where $n = m + 1$. Is G a tree? Give a justification or a counter-example.

b. [3 pts] Let G be an undirected graph with 33 edges. G has 8 vertices of degree 4, 5 vertices of degree 3, 7 vertices of degree 2 and the rest of degree 1. How many vertices have degree 1? Show your steps.

c. [4 pts] Suppose G is an undirected acyclic graph of n vertices and m edges, where $n >> m$. Is G a tree? Is G a forest? How many connected components does G have?

d. [4 pts] If G is a forest of k trees, what is the relationship between the number of vertices (n) and the number of edges (m)? Prove your answer.

7. [8 points] Suppose you are given a table T of size 11 and a set S = {5, 40, 18, 22, 16, 30, 27} to hash into the table, using the hash function $h(k) = k \bmod 11$.

a. [4 pts] Show T after the values from S are entered into it, using chaining.

b. [4 pts] Show T after the values from S are entered into it, using linear probing with the function $h(k,i) = (h(k) + 2i) \bmod 11$.

8. [10 points]

a. [6 pts] You are given 16 balls which are all looked identical. There is one ball which is different from the others in weight. Now you have a balance. Try to identify the different one with the least number of comparisons. Describe your method. How many comparisons are required in your method?

b. [4 pts] You are given k balls which are all looked identical. There is one ball which is different from the others in weight. Now you have a balance. What is the least number of comparisons you need to find out the different one? Prove your answers. [Hint: think about the decision tree.]

9. [15 pts]

a. [5 pts] Let G be an edge-weighted directed graph with source vertex s and let T be a shortest path tree from s. Suppose we add a positive constant p to the cost of every edge in G. Does T remain a shortest path tree from s? Either argue that it does or give a counter-example.

b. [5 pts] Let G be an edge-weighted directed graph with source vertex s and let T be a shortest path tree from s. Suppose we only add a positive constant p to the cost of every edge incident on s in G. Does T remain a shortest path tree from s? Either argue that it does or give a counter-example.

c. [5 pts] Consider a communication network of nodes where node v needs to broadcast a single message to all the other nodes efficiently. Should the message be sent on the minimum spanning tree or the shortest paths from v? Justify your answer and give an example to support your argument. (Note: An intermediate node w can make copies of the message to send on multiple branches of the tree; node v need not send multiple copies to w.)