# 國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 資訊工程研究所　不分組　科目: 線性代數　　共 1 頁　第 1 頁

答題說明：總共有 4 題，第 1 題中有 10 小題，每一小題中各有一敘述，若某一敘述是對的請標示 "T" 且簡單說明為什麼是對的，若是錯的請標示 "F"，並給一反例．每一小題 5 分，沒有說明或是反例不予計分．第 2 至 4 題，請將計算或推導過程寫出，只有答案而沒有過程將予以扣分．

1. True of false, with reason if true and counterexample if false: (50%)

(01) If the entries of matrix $A$ are integers, and det($A$) is 1 or -1, then the entries of $A^{-1}$ are integers. (Here, det($A$) denotes the determinant of matrix $A$.)

(02) If the entries of $A$ and $A^{-1}$ are all integers, then det($A$) is 1 or -1.

(03) Suppose $V$ is a vector space of dimension 7 and $W$ is a subspace of dimension 4. Then, every basis for $W$ can be extended to a basis for $V$ by adding three more vectors, and

(04) every basis for $V$ can be reduced to a basis for $W$ by removing three vectors.

(05) Every invertible matrix can be diagonalized.

(06) Exchanging the rows of a $2 \times 2$ matrix reverses the signs of its eigenvalues.

(07) If vectors $x$ and $y$ are orthogonal, and $P$ is a projection matrix, then $Px$ and $Py$ are orthogonal.

(08) For any two matrices $A$ and $B$ with the same size, rank($A$+$B$) $\leq$ rank($A$) + rank($B$).

(09) For any two square matrices $A$ and $B$, $AB$ and $BA$ have the same set of eigenvalues.

(10) If $A$ is a nonzero square matrix and $A^3$ is the zero matrix, then it is possible that $A$-$I$ is singular. (Here, $I$ represents the identity matrix.)

參考用

2. Let $A$ be a $3 \times 3$ matrix that represents a rotation in $\mathbf{R}^3$.

(a) Describe a method which can find the axis and the angle of the rotation represented by $A$. (8%)

(b) Consider a rotation that takes vector $(x_1, x_2, x_3)$ into vector $(x_2, x_3, x_1)$. Find the matrix that represents this transformation. (7%)

(c) Apply the method in (a) to the matrix you get in (b), and find the axis and the angle of the rotation given in (b). (5%)

3. Let $S$ be the subspace of $\mathbf{R}^4$ containing all vectors $(x_1, x_2, x_3, x_4)$ with $x_1 + x_2 + x_3 + x_4 = 0$ and $x_1 + 2x_2 + 3x_3 + 4x_4 = 0$.

(a) Find two bases for the space $S$ and the space $S^{\perp}$ (the space containing all vectors orthogonal to $S$) respectively. (10%)

(b) Find the projection of vector (0,1,2,7) onto the space $S^{\perp}$. (10%)

4. Find the intersection $V \cap W$ and the sum $V + W$ if

(a) $V$ = null space of a matrix $A$ and $W$ = row space of $A$. (5%)

(b) $V$ = the set of symmetric $3 \times 3$ matrices and $W$ = the set of upper triangular $3 \times 3$ matrices. (5%)