

所別：資訊工程學系碩士班
軟體工程研究所碩士班

科目：離散數學與線性代數

※請務必按照題號次序寫在答案紙上，否則將嚴重失分。

- 1.(25%) Let $S = \{2, 3, 5, 7, 11, 13, 17, 19\}$ be the set of prime numbers less than 20. If A is a subset of S , we can form the sum and product of the elements of A . For example, if $A = \{7, 11, 13\}$, then the associated sum is $7 + 11 + 13 = 31$ and the associated product is $7(11)(13) = 1001$.
- (a) (15%) Use the Pigeon-Hole Principle to show that there are four nonempty subsets of S with the same sum.
- (b) (10%) Are there two nonempty subsets of S with the same product? Explain.
- 2.(10%) (a) (5%) Prove that between every two rational numbers p and q , $p < q$, there is another rational number r that $p < r < q$.
- (b) (5%) Prove that there is an infinite number of rational numbers.
- 3.(15%) Suppose there is a process of validating a group of n items ordered from number 1 to n . Each round every second items can be checked (2^{nd} , 4^{th} , 6^{th} , ...), and the remaining items will be validated with the same process (that every second items of the remaining unchecked items being checked in a round), until only one item left to be checked.
- (a) (6%) Let $f(n)$ be the function to indicate the number of the **last** checked item, give a recursive relation to define $f(n)$.
- (b) (9%) Find the closed form of $f(n)$.

(還有第二頁)

注意：背面有試題

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4.(20%) True and false (每小題答對給 2 分，答錯扣 2 分，不答 0 分，本題總分 ≥ 0)

- (a) The set $\text{Span}\{u, v\}$ is always visualized as a plane.
- (b) The set $\text{Span}\{u, v, w\}$ always contains the origin.
- (c) The subset of a linearly independent vector set is not always linearly independent.
- (d) Sometimes the solution of a nonhomogeneous system contains the origin.
- (e) Scaling and rotation are linear transformation, but translation is not linear transformation.
- (f) Perspective projection is combined from the perspective and projection transformations; the perspective projection is not linear transformation, but perspective transformation is invertible.
- (g) $\det(-A) = -\det A$.
- (h) A linear transformation maps a linearly independent set to another linearly independent set.
- (i) If $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$, then matrix $P = [c_1 \ c_2]^{-1} [b_1 \ b_2]$ satisfies $[x]_C = P[x]_B$.
- (j) The columns of the change-of-coordinates matrix $P_{C \leftarrow B}$ are C -coordinate vectors of the vector in B .

5.(5%) True and false (每小題答對給 1 分，答錯扣 1 分，不答 0 分，本題總分 ≥ 0)The linear system $A_{m \times n} x_{n \times 1} = b_{m \times 1}$ is consistent, if

- (a) The echelon form of A has no row $[0 \ 0 \ \dots \ 0 \ c]$, where $c \neq 0$.
- (b) A has a pivot in every column.
- (c) The columns of A span R^m .
- (d) There is a matrix $n \times m$ D such that $AD = I$.
- (e) $n = m$ and $\text{Nul } A = \{0\}$.

6.(10%) Suppose a sequence of number x_0, x_1, x_2, \dots is determined by the condition that $x_0=1$ and $x_1=2$, and each successive x_n is given by $x_{n+2} = 2x_n - x_{n+1}$, $n \geq 0$. Find a formula for x_n in terms of n .7.(15%) Find the orthogonal project matrix P_W for the subspace W span by the columnspace (image) of the following matrix A , $A = \begin{bmatrix} 1 & 0 & 5 & -3 \\ 0 & 1 & 2 & 4 \\ -1 & -2 & -9 & -5 \\ 1 & 1 & 7 & 1 \end{bmatrix}$.

(後面沒有題目了)