國立中央大學99學年度碩士班考試入學試題卷

所別：資訊工程學系碩士班 不分組(一般生)　科目：離散數學與線性代數　共 二 頁 第 一 頁

軟體工程研究所碩士班 不分組(一般生)　＊請在試卷答案卷（卡）內作答

＊本科考試禁用計算器

【多選題，每題5分，完全答對得5分，答錯得0分，共100分。】

1. Let R be the set of all real number, N be the set of all natural numbers, Z be the set of all integer numbers. Which of the following statements are true?
   (a) Let Q be the set of all integer pairs (n,m), we can prove $|Q|>|N|$.
   (b) The diagonalization method that is used to prove $|R|>|N|$ is a kind of contradiction proof.
   (c) There exists an onto function $f:Z\to Q$.
   (d) There exists a program to determine whether or not any arbitrary program can stop, and this program has to execute with exponential time complexity.
   (e) None of the above.
2. Suppose x and y are integer numbers, and we define the following predicates:
   $D(x,y)$: y is a multiple of x; $E(x)$: x is even; $O(x)$: x is odd
   Which of the following clauses are correct interpretations of the logical statement: $\exists x,y,O(x)\wedge\neg E(y)\to\neg D(x,y)$
   (a) No odd integer can divide any even integer".
   (b) If y is a multiple of x, it is not possible that x is odd and y is not even. .
   (c) It is possible that some not-even number is not a multiple of some odd number..
   (d) Some odd integer is not equal to an even integer.
   (e) none of the above.
3. To analyze the complexity of the following procedure P, We will use the following assumptions: Suppose P and B are both procedures. B take $\theta(m)$ time to compute, where m is the size of input; each statement line in and outside the loop counts 1 step.

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Procedure P( array1[ a_1, a_2, ..., a_n ])
1. if n<3 exit.
2. call B(array1[ a_1, a_2, ..., a_n ])
   declare initially new empty array2, array3;
3. for (i=1 to n)
4. { if ((i mod 3) =0)
5.       insert  a_i  into array2;
6   if ((i mod 3) =1)
7.       insert  a_i  into array3; }
8. call P( array2 );
9. call P( array3 );
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   Suppose n is a multiple of 3, which of the following relations on C_n can describe the complexity of procedure P with respect to problem size n?
   (a) $C_n=2C_{n/3}+\theta(n)$　　(b) $C_n=2C_{n/2}+C_n+\theta(n)$
   (c) $C_n=2C_{n/2}+\frac{3}{2}\theta(n)$　　(d) $C_n=2C_{n/3}+\theta(n)+\theta(1)$
   (e) None of the above.
4. What is the time complexity level of the procedure P in question 3?
   (a) $O(n^{\sqrt{3}})$ (b) $O(n\log n)$ (c) $O(n^{\log n})$ (d) $O(n^2)$ (e) None of the above.
5. Let $\mathbf{A}=\begin{bmatrix}0&0&1&0\\0&1&0&1\\0&0&0&1\\0&0&1&0\end{bmatrix}$ be the matrix to represent a binary relation $\mathbf{R}$ on a four-elements set. Which of the following statements are true?
   (a) There are seven 1's in the matrix that represents the symmetric closure of **R**.
   (b) **R** is a partial ordering relation.
   (c) **R** is reflexive.
   (d) The directed graph of **R** does not have a strongly connected component..
   (e) None of the above.
6. The Towers of Hanoi is a popular puzzle. It consists of three pegs and a number of discs of differing diameters, each with a hole in the center. The discs initially sit on one of the pegs in order of decreasing diameter (smallest at top, largest at bottom), thus forming a triangular tower. The object is to move the tower to one of the other pegs by transferring the discs only to an **adjacent** peg one at a time in such a way that no disc is ever placed upon a smaller one. Solve the puzzle and find the solution for a_n, the number of moves required to transfer n discs from one peg to another.
   (a) When there are n = 2 discs, three moves are required.
   (b) When there are n = 3 discs, thirteen moves are required.
   (c) The recurrence relation is $a_n=2a_{n-1}+1$
   (d) The recurrence relation is $a_n=4a_{n-1}+1$
   (e) The explicit formula for a_n is $a_n=\frac{1}{2}(3^n-1)$
7. You and your buddy return home after a semester at college and are greeted at the airport by your mothers and your buddy's two sisters. Not uncharacteristically, there is a certain amount of hugging! Later, the other five people tell you the number of hugs they got and, curiously, these numbers are all different. Assume that you and your buddy did not hug each other, your mothers did not hug each other, and your buddy's sisters did not hug each other. Assume also that the same two people hugged at most once.
   (a) You hugged three people.
   (b) Your buddy hugged two people.
   (c) Your buddy hugged four people.
   (d) Your mother hugged five people.
   (e) You hugged two people.
8. Which of the following statements are true?
   (a) A Hamiltonian graph contains no proper cycles.
   (b) Every vertex in a Hamiltonian graph has degree 2.
   (c) Every Eulerian graph is Hamiltonian.
   (d) Every Hamiltonian graph is Eulerian.
   (e) A connected graph G has 11 vertices and 53 edges. G is Hamiltonian but not Eulerian.
9. Define $f:A\to B$ by $f(x)=x^2+14x-51$.
   (a) $A=N,\ B=\{b\in Z\mid b\geq-100\}$, f is not onto, but it is one-to-one.
   (b) $A=Z,\ B=\{b\in Z\mid b\geq-100\}$, f is neither onto nor one-to-one.
   (c) $A=R,\ B=\{b\in Z\mid b\geq-100\}$, f is neither onto nor one-to-one.

參考用

注意：背面有試題

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(d) $A = Z$, $B = Z$, f is not onto, but it is one-to-one.
(e) $A = R$, $B = R$. f is not onto, but it is one-to-one.

10. Let $A = \{1, 2, 4, 6, 8, 10\}$, $B = A^2$, and for $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in B, define a relation R by $a\,R\,b$ if and only if $a_1 \le b_1$ and $a_1 + a_2 \le b_1 + b_2$.
(a) R is reflexive and transitive.
(b) R defines an equivalence relation on B.
(c) R defines a partial order on B.
(d) There is no maximum element in this partially ordered set.
(e) This partially ordered set is a lattice.

11. Which of the following are subspaces of R^3?
(a) All vectors of the form $(a, 0, 0)$.
(b) All vectors of the form (a, b, c) where $b=a+c$.
(c) All vectors of the form (a, b, c) where $b=a\times b$
(d) All vectors of the form $(a, 1, 2)$.

12. Suppose that A is a 4×6 matrix. Determine which of the following statements are true?
(a) The rank of A is at most 4.
(b) The rank of A^T is at most 4
(c) The number of general solution of Ax=0 is at most 6.
(d) The nullity of A^T is at most 6.

13. Determine which of the following statements are true?
(a) Given two bases for the same inner product space. There is always a transition matrix from one basis to the other basis
(b) The transition matrix from B to B is always the identity matrix.
(c) Any invertible $n\times n$ matrix is the transition matrix for some pair of bases for R^n.
(d) $\begin{bmatrix}1&2\\1&1\end{bmatrix}$ is the transition matrix from $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ to $\begin{bmatrix}1&2\\2&1\end{bmatrix}$.

14. Indicate which of the following statements are true?
(a) If A is diagonalizable and invertible, then A^{-1} is diagonalizable.
(b) A square matrix with linearly independent column vectors is diagonalizable.
(c) $\begin{bmatrix}3&0&0\\0&2&0\\0&1&2\end{bmatrix}$ is diagonalizable.
(d) If matrix A is diagonalizable, then there is a unique matrix P such that $P^{-1}AP$ is a diagonal matrix

15. Suppose that the characteristic polynomial of matrix A is found to be $P=(\lambda-1)(\lambda-4)^2(\lambda-5)^3$. Indicate which of the following statements are true?
(a) The dimension of the eigenspace corresponding to $\lambda=1$ is 1.
(b) If A is diagnosable, the dimensions of the eigenspaces corresponding to $\lambda=1$, 4, and 5 must be 1, 2, and 3, respectively.
(c) If $\{v_1, v_2, v_3\}$ is a linearly independent set of eigenvectors of A all of which correspond to the same eigenvalue of A. Then the eigenvalue must be 4.
(d) If B and C are two $n\times n$ square matrices, then BC and CB have the same set of eigenvalues.

16. Let $A = \begin{bmatrix}\lambda-5&0&0&0\\0&\lambda&1&2\\3&1&\lambda-1&3\\2&4&3&5\end{bmatrix}$. Find all values of λ for which matrix A is not invertible.

(a) $\lambda=5$　　(b) $\lambda=5/7$
(c) $\lambda=3$　　(d) $\lambda=3/14$

17. Let $B = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right\}$ and $\hat{B} = \left\{\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$. Suppose that $[T]_B = \begin{bmatrix}4&-2\\2&3\end{bmatrix}$ is the linear transformation relative to the basis B. Then the corresponding matrix $[T]_{\hat{B}}$ relative to the basis $\hat{B}$ is

(a) $\begin{bmatrix}1&-2\\5&6\end{bmatrix}$　　(b) $\begin{bmatrix}6&4\\-1&2\end{bmatrix}$

(c) $\begin{bmatrix}-2&3\\6&1\end{bmatrix}$　　(d) $\begin{bmatrix}6&-1\\-4&2\end{bmatrix}$

18. Suppose matrices $A=\begin{bmatrix}1&0\\-1&2\end{bmatrix}$ and $B=\begin{bmatrix}0&-2\\1&3\end{bmatrix}$. Indicate which of the following statements are true?

(a) $B^{10}=\begin{bmatrix}2-2^{10}&2-2^{11}\\-1+2^{10}&-1+2^{11}\end{bmatrix}$　　(b) $B^{10}=\begin{bmatrix}2-2^{11}&2-2^{10}\\-1+2^{11}&-1+2^{10}\end{bmatrix}$

(c) $A^{10}=\begin{bmatrix}1&0\\1-2^{10}&1-2^{10}\end{bmatrix}$　　(d) $A^{10}=\begin{bmatrix}1&0\\1-2^{10}&2^{10}\end{bmatrix}$

19. Let CS(A) denote then column space of matrix A and RS(A) denote the row space of matrix A. Indicate which of the following statements are true?
(a) CS(AB) $\subseteq$ CS(A)　　(b) RS(AB) $\subseteq$ RS(B)
(c) CS($A+B$) $\subseteq$ CS(A)+CS(B)　　(d) RS($A+B$) $\subseteq$ RS(A)+RS(B)

20. A linear transformation $T:R^2\to R^2$ such that $T(1, 1) = (1, 0, 1)$ and $T(2, 3) = (1, -1, 4)$.
Indicate which of the following statements are true?
(a) $T(14, 19) = (9, -5, 24)$　　(b) $T(3, 4) = (2, -1, 5)$
(c) $T(4, 6) = (2, -2, 8)$　　(d) $T(0,1) = (0, -2, 6)$

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