國立中央大學108學年度碩士班考試入學試題

所別： 通訊工程學系碩士班 不分組(一般生)　　

科目： 工程數學(線性代數、機率)

本科考試禁用計算器

計算題　　＊計算題需計算過程，無計算過程者不予計分

1. (10%) Let A and B be 3×3 matrices with $\det(A) = 5$ and $\det(B) = -6$. Find the value of:

   (1) (5%) $\det(3AB)$

   (2) (5%) $\det(A^{-1}B)$

2. (15%) Let x_1, x_2, and x_3 be linearly independent vectors in $\mathbb{R}^n$ and let

   $$y_1 = x_1 + x_2,\ y_2 = 2x_3 - 2x_1,\ y_3 = 3x_2 + 3x_3$$

   Are y_1, y_2, and y_3 linearly independent? If it is true, please explain your reason; otherwise, please show that there are nontrivial linear combinations of y_1, y_2, y_3 that equal 0.

3. (15%) Let A be an $m \times n$ matrix whose rank is equal to n and let $\mathbf{c}$ and $\mathbf{d}$ are column vectors in $\mathbb{R}^n$. If $A\mathbf{c} = A\mathbf{d}$, does this imply that $\mathbf{c}$ must be equal to $\mathbf{d}$? What if the rank of A is less than n? Explain your answers.

4. (10%) If A is a singular matrix, is it true that A has 0 as an eigenvalue? Please explain your answer.

5. (40%) If X and Y are random variables with joint probability density function (PDF) $f_{X,Y}(x, y)$, given by

   $$f_{X,Y}(x,y) = \begin{cases} \lambda\mu e^{-(\lambda x + \mu y)}, & x \geq 0, y \geq 0 \\ 0, & otherwise \end{cases}$$

   (1) (6%) Please find the marginal probability density function $f_X(x)$.

   (2) (6%) Please find the mean of the random variable X.

   (3) (6%) Are the random variables X and Y independent? Please explain your answer.

   (4) (6%) Are the random variables X and Y uncorrelated? Please explain your answer.

   (5) (10%) Please find the cumulative density function (CDF) of a random variable $W = Y/X$.

注意：背面有試題

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(6) (6%) Please find the PDF of the random variable $W = Y/X$.

6. (10%) In a communication system, the number of packet arrivals is modeled as a Poisson random variable. X is a Poisson (α) random variable if the probability mass function (PMF) of X has the form:

$$P_X(x) = \begin{cases} \dfrac{\alpha^x e^{-\alpha}}{x!}, & x = 0,1,2,\ldots \\ 0, & otherwise \end{cases}$$

The communication system has on average one arrived packet per second.

(a) (5%) Please elaborate on the meaning of the parameter α in the PMF of X.

(b) (5%) What is the probability that there are no packets arrived from the time $t = 2$ seconds to the time $t = 3$ seconds?

注意:背面有試題