1. (20%) The transmitted FM signal is $x_c(t) = A_c \cos(2\pi f_c t + 2\pi f_d \int^t m(\alpha)d\alpha)$ where $m(t)$ is the lowpass message signal with bandwidth W and the peak value of $|m(t)|$ is unity. The received FM signal is $x_r(t) = x_c(t) + n(t)$ where $n(t)$ is the AWGN with double-sided PSD $N_0/2$.
(a) (3%) What is the bandwidth of $x_c(t)$?
(b) (4%) Plot the block diagram of the receiver including the block "discriminator". The transfer functions of filters should be plotted.
(c) (10%) Assume that the power of $x_c(t)$ is significantly larger than the noise power at the output of the predetection filter. Derive the signal-to-noise ratio (SNR) of the output signal at the receiver.
(d) (3%) Does larger f_d always imply larger SNR in (c)? Explain your viewpoint.

2. (14%) At the transmitter, during $0 \le t \le T_s$, one input data bit chooses between $x_1(t) = \cos\omega_c t + 3\sin\omega_c t$ and $x_2(t) = -3\cos\omega_c t - \sin\omega_c t$. The received signal is $x_r(t) = x_i(t) + n(t)$ where $n(t)$ is the AWGN with double-sided PSD $N_0/2$ and $i \in \{1, 2\}$. Assume that the coherent maximum-likelihood detector is used at the receiver.
(a) (4%) Find the average energy per bit E_b.
(b) (10%) Derive the error probability in terms of E_b/N_0.

3. (16%) Consider the signal

$$s_i(t) = A_i \cos\omega_i t, \qquad 0 \le t \le T_s, \qquad i = 0, 1, 2, \cdots, 7$$

where $\cos\omega_i t$ are orthogonal over the interval $[0, T_s]$. Assume that the coherent maximum-likelihood detector is used at the receiver. For the following two cases, find the (approximated) symbol error probabilities in terms of E_s/N_0 over the AWGN channel with double-sided PSD $N_0/2$ where E_s denotes the average energy per symbol.
(a) (8%) Eight data bits $b_0, b_1, \cdots, b_7$ are conveyed by $A_i = \begin{cases} A & \text{if } b_i = 0 \\ -A & \text{if } b_i = 1 \end{cases}$. The transmitted signal is $x(t) = \sum_{i=0}^{7} s_i(t)$.
(b) (8%) Three data bits b_0, b_1, b_2 select $s_i(t)$ and one data bit b_3 chooses A_i. In other words, the transmitted signal is $x(t) = s_k(t)$ with $A_k = \begin{cases} A & \text{if } b_3 = 0 \\ -A & \text{if } b_3 = 1 \end{cases}$ where $k = b_0 + b_1 \times 2 + b_2 \times 4$.

4. (20%) Consider a double sideband-suppressed carrier modulation signal with a random phase, defined as

$$y(t) = A \cdot m(t) \cdot \cos(2\pi f_c t + \theta) \tag{1}$$

where A and f_c are constants and θ is a uniformly distributed random variable over the interval $[0, 2\pi]$, given by

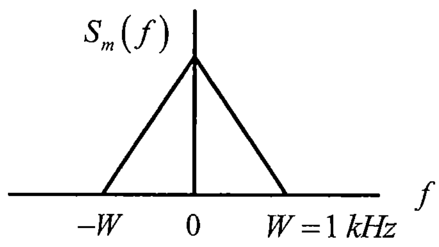
$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \le \theta \le 2\pi \\ 0, & \text{elsewhere} \end{cases} \tag{2}$$

Assume that the autocorrelation function and power spectral density of the signal $m(t)$ are $R_m(\tau)$ and $S_m(f)$, respectively.

(a) (5%) Find the autocorrelation function of $y(t)$.

(b) (5%) Find the power spectral density of $y(t)$.

(c) (5%) Assume $S_m(f)$ is given by



What is the lowest carrier frequency f_c for avoiding sideband overlap in the power spectral density of the signal $y(t)$?

(d) (5%) Plot the block diagram of the Costas receiver for demodulating $m(t)$.

5. (30%) The received signal is defined by

$$x(t) = s_i(t) + w(t), \quad 0 \le t \le 2T, \quad i = 1, 2$$

where $w(t)$ is white Gaussian noise of zero mean and power spectral density $\frac{N_0}{2}$. The signals $s_1(t)$ and $s_2(t)$ are given as

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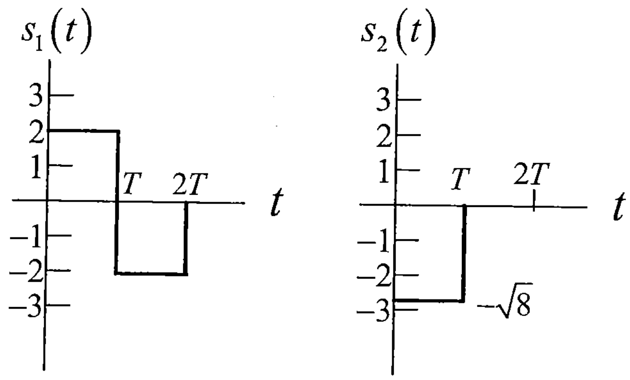

(a) (5%) Find a set of orthonormal basis functions to represent the signals $s_1(t)$ and $s_2(t)$.

(b) (5%) Based on (a), construct the signal constellation for $s_1(t)$ and $s_2(t)$.

(c) (5%) Plot the block diagram of the correlation receiver for decoding $x(t)$.

(d) (5%) If $s_1(t)$ and $s_2(t)$ are transmitted with an equal probability, i.e., $P(s_1(t)) = P(s_2(t)) = \frac{1}{2}$, find the optimal decision rule for the correlation receiver.

(e) (5%) Assume $P(s_1(t)) = P(s_2(t)) = \frac{1}{2}$. Modify the signal constellation in (b) to achieve the minimum average error probability but preserve the same energy of the constellation points.

(f) (5%) Based on (e), find the average error probability.