國立中央大學 107 學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 系統組(一般生)　　共 2 頁　第 1 頁

科目： 工程數學(含程式設計)

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

1. Solutions for ordinary differential equations (ODEs) (25%)

(a) Find the solution for the ODE $y' = (x+y-2)^2, y(0) = 2$. (Hint: set $v = (x+y-2)$) **(7%)**

(b) Find the solution for the 2nd-order ODE $x^2y'' - xy' + y = 0, y(1) = 1.5, y'(1) = 0.25$. **(8%)**

(c) Find the solution of the initial value problem
$y''+3y'+2y = 10[\sin t + \delta(t-1)]$, $y(0) = 1$, $y'(0) = -1$. $\delta(t-1)$ is Dirac delta function. **(10%)**

2. Vector analysis and Linear algebra (25%)

(a) Please find the parametric equations of streamline through *(−1, 6, 2)* for the vector **F***(x, y, z)*=x^2**i**+2*y***j**−1**k** (*x* and *y* are not zero) using the equations $\frac{dx}{x^2} = \frac{dy}{2y} = \frac{dz}{-1}$ **(10%)**

(b) Please solve the following nonhomogeneous systems of ODEs by evaluating (i) matrix form, $J' = AJ + g$ **(2%)**; (ii) eigenvalues and eigenvectors of matrix A **(5%)**; (iii) the corresponding homogeneous and nonhomogeneous solutions **(8%)**

$$\begin{cases} I_1' = -4I_1 + 4I_2 + 12 \\ I_2' = -1.6I_1 + 1.2I_2 + 4.8 \end{cases}$$

3. Laplace transform / Fourier analysis (25%)

The Fourier transform pairs of two time signals $f(t) \leftrightarrow F(\omega)$ and $g(t) \leftrightarrow G(\omega)$ denote $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$ and $G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt$, where t and ω are time and angular frequency, respectively. It's known that two properties of the Fourier transform are called (i) the time shift theorem, that is $f(t-t_0) \leftrightarrow F(\omega)e^{-j\omega t_0}$, and (ii) the convolution theorem, that is $f(t)*g(t) \leftrightarrow F(\omega)G(\omega)$, where $f(t)*g(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$.

(a) Here *f(t)* is a time signal.

(i) **(6%)** Derive (or prove) the Fourier transform of *f(t-t₀)* to be $F(\omega)e^{-j\omega t_0}$.

(ii) **(3%)** If a specific time signal was defined as $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2-t, & 1 \le t < 2 \end{cases}$. Sketch *f(t)* and *f(t-1)*.

(iii) **(3%)** Give the physical meaning of (or give interpretation to) *f(t)* and *f(t-1)* in both the time domain and the frequency domain ($F(\omega)e^{-j\omega t_0}$).

(b) Here *f(t)* and *g(t)* are both time signals.

(i) **(10%)** Derive (or prove) the Fourier transform of *f(t)*g(t)* to be *F(ω)G(ω)*.

(ii) **(3%)** If now *f(t)* is the input guided into a system with the impulse response function *g(t)*, please give the interpretation to the convolution theorem. (To make a sketch for explanation is helpful.)

注意:背面有試題

參考用

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所別： 機械工程學系 碩士班 系統組(一般生)　　共二頁 第二頁

科目： 工程數學(含程式設計)

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4. Programming (25%)

(a)**(10%)** An integer is said to be a perfect number if the sum of its divisors, including 1 (but not the number itself), is equal to the number. For example, 6 is a perfect number, because 6 = 1 + 2 + 3. Write a function **isPerfect** that determines whether parameter number is a perfect number. The code is limit to **C**, **C++**, **Visual Basic** or **Fortran** programing language, and please state before your answer. All variables declare to integers.

(b)**(15%)** A prime integer is any integer greater than 1 that can be divided evenly only by itself and 1. The Sieve of Eratosthenes is a method of finding prime numbers. It works as follows:

a) Create an array with all elements initialized to 1 (true). Array elements with prime subscripts will remain 1. All other array elements will eventually be set to zero.

b) Starting with array subscript 2 (subscript 1 is not prime), every time an array element is found whose value is 1, loop through the remainder of the array and set to zero every element whose subscript is a multiple of the subscript for the element with value 1. For array subscript 2, elements which are greater than 2 and multiples of 2 in the array will be set to zero (subscripts 4, 6, 8, 10, and so on.).

When this process is complete, the array elements that are still set to 1 indicate that the subscript is a prime number. Write a program that uses an array of 1000 elements to determine and print the prime numbers between 1 and 999. Ignore element 0 of the array. The code is limit to **C**, **C++**, **Visual Basic** or **Fortran** programing language, and please state before your answer. All variables declare to integers.

參考用

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