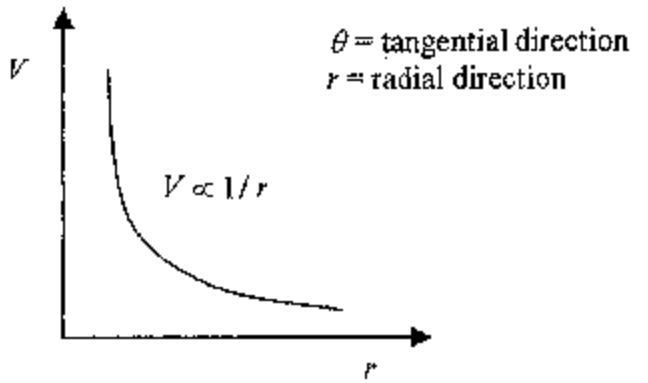
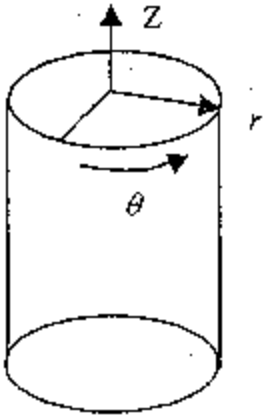
1. Air flowing into a 2-ft-square duct with a uniform velocity of 10 ft/s and a zero pressure ($p_{inlet} = 0$) forms a boundary layer on the walls. The density of air is 2.38×10^{-3} slug/ft^3. For this flow the boundary layer displacement thickness is given by $\delta^* = 0.007x^{1/2}$ where δ^* and x are in feet. (20%)
   (a) Determine the velocity U = U(x) of the air within the duct but outside of the boundary layer.
   (b) Determine the variation of pressure along the flow direction.
   (c) If a uniform velocity must be maintained outside the boundary layer, it is necessary to increases the cross-sectional size of the duct. Determine the variation of duct size along the flow direction.

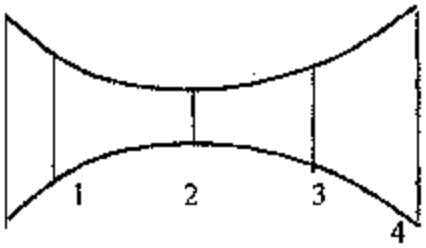
2. Osbone Reynolds studied the flow behavior in a pipe. He discovered that the dimensionless parameter (Reynolds number) is a criterion by which the state of the flow may be determined. (15%)
   (a) What is the definition of the Reynolds number? What is the physical interpretation of the Reynolds number?
   (b) Describe the nature of the pipe flow based on the Reynolds number.
   (c) Describe the evolution of the velocity distribution along the pipe.
   (d) Describe the structure of the turbulent velocity based on the shear stress.

3. Consider rotating a rod in a cylindrical tank until the tangential velocity V of water along the radial direction is $V(r)=c/r$, where c is a constant. The tangential velocity is the only non-zero velocity. You can assume that the water is inviscid and incompressible.
   (a) Can you recognize this flowfield according to the particular velocity distribution, how do you call this flow? (5%)
   (b) Explain and derive the static and total pressure expression along the radial and tangential direction. (10%)





4. (a) List the necessary conditions for a normal shock appears inside the divergent-convergent nozzle. (9%)
   (b) What is the possible position of the normal shock inside the nozzle: 1). upstream the throat, 2). throat, 3). downstream the throat, 4). at the exit? Please briefly explain your answer. (6%)



5. Consider the steady flow of a frictionless fluid in a gravitational field for which the body force is inversely proportional to the square of the distance from a fixed central point. Assume that the density can be expressed as a function of pressure in the form $\rho = \rho_o (p/p_o)^a$, where a is a constant different from unity. Derive the Bernoulli equation in this system for two points on a streamline. (10%)

6. A sphere of 0.25-in diameter is dropped into water with kinematic viscosity $\nu = 10^{-6}$ m^2/s. After an initial period of acceleration it is observed to sink steadily at a velocity of 0. 5 m/s. Determine the ratio of the density of the sphere to that of water. Note that the drag coefficient c_d for a sphere is about 0.4 at $1.5\text{x}10^3 < \text{Re} < 10\text{x}10^3$ and 0.5 at $3\text{x}10^4 < \text{Re} < 5\text{x}10^4$. (10%)

7. A hurricane can be visualized as a planar incompressible flow consisting of a rotating circular core surrounded by a potential flow. A particular hurricane has a core of radius 40m and air is sucked into this core at a volume flow rate per meter depth perpendicular to the diagram of 5000 m^3/s. Furthermore, the pressure difference between the air far away from the hurricane and the air at the edge of the core is 1.5 kPa. The velocity of the air far from the core is assumed to be negligible. The density of the air is assumed uniform and constant at 1.2 kg/m^3. Find the angular rate of rotation of the hurricane. (15%)