所別：**機械工程學系碩士班 甲組(固力與設計)** 科目：**工程數學**
乙組(製造與材料)
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能源工程研究所碩士班

Ordinary Differential Equation (33 %)

1. Assuming that the radium decomposes at a rate proportional to the amount present, in how many years will half the original amount be lost if 10% disappears in 263 years. (8%)

2. (a) Show that $y_1 = x$ and $y_2 = x^2$ are both linearly independent solutions of $x^2 y'' - 2xy' + 2y = 0$
    (4%)

    (b) Find the particular solution for which $y(1) = 3$ and $y'(0) = 5$. (4%)

3. Compute by direct evaluation of the integral the Laplace transform $L[f(x)]$

    (a) $f(x) = \sin 3x$ (4%)

    (b) $f(x) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & t > 1 \end{cases}$ (4%)

4. Show that for the hypergeometric series $F(a,b,c,x)$,

    $$\frac{dF(a,b,c,x)}{dx} = \frac{ab}{c} F(a+1,b+1,c+1,x) \qquad (9\%)$$

    (Hint: the hypergeometric series can be expressed as follows

    $$F(a,b,c,x) = 1 + \sum_{n=1}^{\infty} \frac{a(a+1)\cdots(a+n-1)b(b+1)\cdots(b+n-1)}{n!c(c+1)\cdots(c+n-1)} x^n)$$

Linear Algebra & Vector Calculus (33 %)

5. Find the surface integrate $\iint_s \mathbf{F} \cdot \mathbf{n} dA$, when $\mathbf{F} = [x^2, 0, 2y^2]$，S is the portion of the plane

    $3x + 2y + z = 6$ in the first octant $(x \ge 0, y \ge 0, z \ge 0)$. (15%)

6. If $\mathbf{F} = \frac{2x}{x^2+y^2}\mathbf{i} + \frac{2y}{x^2+y^2}\mathbf{j}$, $C_1 : (x-2)^2 + (y-1)^2 = 1, \mathbf{r} = x\mathbf{i} + y\mathbf{j}$, find the integral

    $\oint_{C_1} \mathbf{F} \cdot d\mathbf{r}$ counterclockwise around C_1. (8%)

7. Consider the matrix $\mathbf{A}$, Determine matrices $\mathbf{Q}$ and $\mathbf{D}$ such that $\mathbf{Q^{-1}AQ=D}$ is diagonal. (10%)

    $$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

注意：背面有試題

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Fourier Analysis, Partial Differential Equation and Complex Analysis (34 %)

8.　Find the Fourier series of

$$f(x) = \begin{cases} 0 & -2 < x < 0 \\ x & 0 \le x < 1 \\ 1 & 1 \le x < 2 \end{cases} \quad (10\%)$$

9.　(a) Use separation of variables to find the product solutions of

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad k > 0 \quad (4\%)$$

　　(b) Find the temperature $u(x, t)$ in a rod of length 2 if the initial temperature is

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$$ throughout and if the ends $x = 0$ and $x = 2$ are insulated. (10%)

10.　(a) Use a Laurent series to find the indicated residue.

$$f(z) = \frac{e^{-z}}{(z-2)^2} \quad ; \quad \text{Re}\,s(f(z),2) \quad (5\%)$$

　　(b) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{\cos 2x}{x^2+1}\,dx \quad (5\%)$