1) Let $y(t)$ be a solution of the differential equation $y'' + 4y' + 13y = 0$. It satisfies the initial conditions $y(0) = 0$ and $y'(0) = 3$. Please compute $y(\frac{\pi}{6})$.　　(30 points)

2) A curve in the (x,y) plane is described by the equation $r = \sin\theta$. Here r and θ are the polar coordinates. The curve connects two points A and B. The coordinates of A are $(r,\theta) = (0,0)$ and the coordinates of B are $(r,\theta) = (1, \frac{\pi}{2})$. Please compute the length of this curve from A to B.

(30 points)

3) Consider the matrix $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$. It can be expressed as $A = RU$. Here R is an orthonormal matrix which satisfies the relation $R^T R = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. We also know that U is a symmetric matrix. If the matrix R can be written as $R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$, please compute the value of r_{11}.　　(20 points)

4) The period P of a periodic function $f(x)$ is $P = 1$. In the interval $0 < x < 1$ the function can be expressed as $f(x) = 1 - 2x$. We can calculate the Fourier series of this periodic function. After this Fourier series has been constructed we can make use of the discontinuity of $f(x)$ to find out the Fourier series of the Delta function $\delta(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2n\pi x)$ in the interval $0 < x < 1$.

Please compute the coefficient a_n.　　(20 points)