國立中央大學109學年度碩士班考試入學試題

所別： 地球科學學系地球物理 碩士班 不分組(一般生)
地球科學學系地球物理 碩士班 不分組(在職生)

科目： 微積分

本科考試禁用計算器

＊請在答案卷(卡)內作答

作答時須列出完整計算過程

1. (a) $y = x^x$, $\frac{dy}{dx}$ =? (5%)

   (b) $\lim_{x\to 0}\frac{x-tanx}{x-sinx}$ (5%)

2. (a) $\int x^3 sin5x dx$ (5%)

   (b) $\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{1}{a}e^{i\omega t}\right)dt$ =? (5%)

3. (10%) Find the general solution.

$$y' + ky = e^{-kx}$$

4. (10%) Find the initial value problem.

$$y'' + y' + 0.25y = 0, \quad y(0) = 3.0, \quad y'(0) = -3.5$$

5. (5%) (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 7 \\ 6 & 6 \end{bmatrix}$$

(5%)(b) Find the inverse of A matrix above.

6. (10%) "Fermat's principle" states that the path taken between two points by a ray of light is the least-time path. Derive Snell's law using "Fermat's principle".

注意：背面有試題

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7. $e^{i(\omega t-\vec{k}\cdot\vec{x})}$ represents a propagating plane wave in 3-D space, where $\vec{k}=k_x\hat{i}+k_y\hat{j}+k_z\hat{k}$ is the wave number vector indicating the direction of propagation and $\vec{x}=x\hat{i}+y\hat{j}+z\hat{k}$ is the position vector.

(a) Given a scalar potential $\phi(\vec{x},t)=e^{i(\omega t-\vec{k}\cdot\vec{x})}$, show that the displacement of its gradient is parallel to the direction of propagation. (5%)

(b) Given a vector potential $\vec{\gamma}(\vec{x},t)=\left(A_x,A_y,A_z\right)e^{i(\omega t-\vec{k}\cdot\vec{x})}$, show that the displacement of its curl is perpendicular to the direction of propagation. (5%)

8. (10%) Find the **odd** periodic expansions of the function (half-range expansion)

$$f(x)=\begin{cases}\dfrac{2k}{L}x & \text{if} \quad 0<x<\dfrac{L}{2}\\ \dfrac{2k}{L}(L-x) & \text{if} \quad \dfrac{L}{2}<x<L.\end{cases}$$

9. (10%) Use the method of separating variables to solve the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$, for the vibrations of an elastic string of length L.
The boundary conditions are $u(0,t)=0, u(L,t)=0$ for all t.
The initial conditions are $u(x,0)=f(x), u_t(x,t)|_{t=0}=g(x)$.

10.(10%) Use Laplace transform to solve $\frac{\partial^2 w(x,t)}{\partial t^2}=c^2\frac{\partial^2 w(x,t)}{\partial x^2}$, with two boundary conditions (1) $w(0,t)=f(t)=\begin{cases}sint, & if\ 0\le t\le 2\pi\\ 0 & otherwise\end{cases}$,

(2) $\lim_{x\to\infty} w(x,t)=0 \quad (t\ge 0)$, and two initial conditions (1) $w(x,0)=0$,

(2) $\frac{\partial w}{\partial t}|_{t=0}=0$.

注意：背面有試題