作答時須列出完整計算過程

1. (a) $tan\theta = \frac{y}{x}$, $\frac{\partial\theta}{\partial x} =?$ (5%)

   (b) $\lim_{x\to 0}(cscx - cotx) =?$ (5%)

2. (a) $\int x^3 cos7x dx$ (5%)

   (b) $\int_{-\infty}^{\infty} x^2 \, dx =?$ (5%)

3. (10%) Find the general solution.
$$y'' + 3y' + 2y = 12x^2$$

4. (10%) Find the eigenvalues and eigenvectors of the matrix
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix}$$

5. (10%) "Fermat's principle" states that the path taken between two points by a ray of light is the least-time path. Derive Snell's law using "Fermat's principle".

6. (10%) Evaluate $\int_C \vec{F}(\vec{r}) \, d\vec{r}$ counterclockwise around the boundary C of the region R by Green's theorem, where
$\vec{F} = [y, -x]$, C the circle $x^2 + y^2 = \frac{1}{4}$.

注意:背面有試題

國立中央大學 110 學年度碩士班考試入學試題

所別： 地球科學學系地球物理 碩士班 不分組(一般生)
　　　　地球科學學系地球物理 碩士班 不分組(在職生)
科目： 微積分
本科考試禁用計算器

共 2 頁　第 2 頁

＊請在答案卷(卡)內作答

7. (10%) Find the **even** periodic expansions of the function (half-range expansion)

$$f(x) = \begin{cases} \dfrac{2k}{L}x & \text{if} \quad 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(L-x) & \text{if} \quad \dfrac{L}{2} < x < L. \end{cases}$$

8. (10%) Use the method of separating variables to solve the one-dimensional wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$, for the vibrations of an elastic string of length L.

The boundary conditions are $u(0,t) = 0, u(L,t) = 0$ for all t.
The initial conditions are $u(x,0) = f(x), u_t(x,t)|_{t=0} = g(x)$.

9. (10%) Fill out the table for Laplace Transform

	f(t)	F(s)		f(t)	F(s)
1	t		6	$\sin\omega t$	
2	t^n		7	$\cosh at$	
3	t^a		8	$\sinh at$	
4	e^{at}		9	$e^{at}\cos\omega t$	
5	$\cos\omega t$		10	$e^{at}\sin\omega t$	

10. (10%) Show that the line integrals $\int_C \vec{F}(\vec{r})d\vec{r} = \int_C (F_1 dx + F_2 dy + F_3 dz)$ in a domain D in space is path independent if and only if $\vec{F}$ is the gradient of some function f in D.

注意:背面有試題