所別：地球物理研究所碩士班 不分組 科目：微積分

1. Calculate the limits:

(a) $\lim_{t\to 0}\dfrac{\sin(\tan(t))}{\sin(t)}$ (5%) (b) $\lim_{t\to 0} t\sin(\frac{1}{t})$ (5%)

2. The addition formulas for the sine and cosine are given as the following:

$$\begin{cases}\sin(A\pm B)=\sin(A)\cos(B)\pm\cos(A)\sin(B)\\ \cos(A\pm B)=\cos(A)\cos(B)\mp\sin(A)\sin(B)\end{cases}$$

(a) Prove that $\int_0^{2\pi}\sin(nx)dx=\int_0^{2\pi}\cos(nx)dx=0$ for all integers $n\neq 0$. (5%)

(b) Use (a) and the addition formulas for the sine and cosine to establish the following formulas, valid for integers m and n, $m^2\neq n^2$:

$$\begin{cases}\int_0^{2\pi}\sin(nx)\cos(mx)dx=\int_0^{2\pi}\sin(nx)\sin(mx)dx=\int_0^{2\pi}\cos(nx)\cos(mx)dx=0\\ \int_0^{2\pi}\sin^2(nx)dx=\int_0^{2\pi}\cos^2(nx)dx=\pi \quad \text{if } n\neq 0\end{cases}$$ (10%)

3. Let $\begin{cases}s(t)=\sin(t)/t & \text{for } t\neq 0\\ s(0)=1\end{cases}$ and define $T(x)=\int_0^{\infty}s(t)dt$.

(a) Prove that $s(t)$ is continuous at $t=0$. (5%)

(b) Prove that the function $f(x)=x\,T(x)$ satisfies the differential equation $xy'-y=x\sin(x)$. (10%)

4. If α is a real number and n is a nonnegative integer, the binomial coefficient $\binom{\alpha}{n}$ is defined by the equation $\binom{\alpha}{n}=\dfrac{\alpha(\alpha-1)(\alpha-2)......(\alpha-n+1)}{n!}$.

(a) When $\alpha=-1/2$, show that $\binom{\alpha}{1}=-\frac{1}{2}$, $\binom{\alpha}{2}=\frac{3}{8}$, $\binom{\alpha}{3}=-\frac{5}{16}$, $\binom{\alpha}{4}=\frac{35}{128}$. (5%)

(b) Let $a_n=(-1)^n\binom{-1/2}{n}$. Prove that $a_n>0$ and $a_n>a_{n+1}$. (10%)

參考用

5. Let $f(x)=(2+x^2)^{5/2}$. Determine the coefficients a_0, a_1, a_2, a_3 in the Taylor's series

注意：背面有試題

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generated by f at 0. (10%)

6. Prove that for any motion the dot product of the velocity and acceleration vectors is half the derivative of the square of the speed. (10%)

7. The thickness of a bottomset bed at the foot of a delta can often be well approximated by the expression

$$t = t_0 \exp(-x/x_0) \qquad \text{(EQ1)}$$

where t is thickness, x is distance from the bottomset bed start and t_0 and x_0 are constants.

(a) Imagine approximating this sedimentary bed in cross-section by a series of rectangles of height t_i and width Δx (Figure 1). What is the area of each rectangle? (5%)

(b) Now write down an approximate sum for the cross-sectional area of the entire bottomset bed with a series of N rectangles of equal width Δx but different height. (5%)

(c) By considering the limiting case of an infinite number of infinitely thick rectangles, write down and evaluate an integral equation giving the total cross-sectional area. (10%)

(d) If the present-day rate of sediment supply is 10 m^2/year, x_0 = 5 km and t_0 = 1 m, estimate the time taken to form the bed assuming the sediment supply rate has not altered through time. (5%)

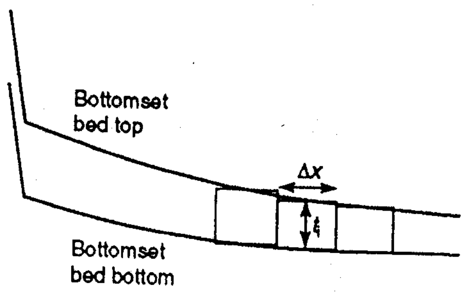

Figure 1. Approximating the bottomset bed by a series of rectangular elements of thickness t_i ανδ ωιδτη Δx.