所別：地球物理研究所碩士班 一般生 科目：應用數學

1. Show the following equations are exact and solve it?

   a. $(xy^2 - y)dx + (x^2y - x)dy = 0$ **(5 pts)**

   b. $(3x - 6xy)dx + (3y^2 - 3x^2)dy = 0$ **(5 pts)**

2. Find a general (or particular) solution of each of the following equations:

   a. $y' - \frac{3}{x}y + x = 0$ **(10 pts)**

   b. $y' + 2xy - x = 0, \quad y(0) = \frac{3}{2}$ **(10 pts)**

3. Find complete solutions of the following equations:

   a. $(D^3 - 2D^2 - 3D + 10)y = 0$ **(10 pts)**

   b. $y'' + 6y' + 9y = e^{-3x}$ **(10 pts)**

4. **Find** the Fourier series of the periodic function whose definition in one period is

$$f(t) = \begin{cases} a & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases}$$ **(10 pts)**

5. Find the half-range cosine expansion of the following function:

$$f(t) = t^2, \quad 0 < t < 1$$ **(10 pts)**

6. Find the Fourier integral representation of the following function:

$$f(t) = \begin{cases} 1 - t & 0 < t < 1 \\ 0 & others \end{cases};$$ **(10 pts)**

7. If $F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$; *and* $f(t) = \int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega$ is the Fourier transform pair, show that

   a. if ω_0 is a real constant, then the Fourier transform of

   $e^{i\omega_0 t}f(t)$ is $F(\omega - \omega_0)$ **(10 pts)**

   b. the Fourier transform of $f'(t)$ is $i\omega F(\omega)$ **(10 pts)**