國立中央大學九十學年度碩士班研究生入學試題卷

所別: 太空科學研究所　不分組　科目: 近代物理　共 1 頁 第 1 頁

1. A photon collides with a stationary electron to produce an electron-positron pair, $\hbar\omega + e^- \rightarrow e^- + (e^- + e^+)$.
   Find the minimum energy for the photon in terms of electron rest energy. (5%)

2. An observer moves at a speed u away from a source.
   (a) Derive the formulas for the Doppler effect both in light and in sound. (10%)
   (b) When $u << c$, are the formulas in (a) reduced to the same form? (5%)

3. A metal surface illuminated by $5\times10^{14}\,Hz$ light emits electrons whose maximum kinetic energy is $0.5eV$. The same surface illuminated by $8.6\times10^{14}\,Hz$ light emits electrons whose maximum kinetic energy is $2eV$.
   Find Planck's constant and the work function of the surface. (10%)

4. A particle of mass m in a box l wide,
   (a) Find the uncertainty in position for the particle in the ground state and first excited state. (10%)
   (b) Find the uncertainty in momentum for the particle in the ground state and first excited state. (10%)
   (c) Find the limit of the uncertainty in position and in momentum for large quantum number. (10%)
   (d) Calculate the expectation value of the product of momentum and position $< px >$ for the particle in the first excited state. Is it the same as the expectation value $< xp >$? (10%)

5. From the quantum theory of the hydrogen atom, what are the angles between its orbital angular momentum vector $\bar{L}$ and the z-axis for orbital quantum number $l = 1$? and for $l = 2$? (10%)

6. Consider a system of three electrons in states specified by quantum numbers n_1, n_2 and n_3. Let the one-electron states be $\psi_{n_1}(r_1)$, $\psi_{n_2}(r_2)$ and $\psi_{n_3}(r_3)$.
   By using the one-electron states, construct a state of the system that is anti-symmetric with respect to an interchange of any two electrons. (5%)

7. A gas of 4 particles, each of which can occupy a state of energy $E_n = n\varepsilon$, where n is an integer (0,1,2...). The total energy of the system is $E = 9\varepsilon$.
   List all possible macrostates and determine the number of microstates associated with each macrostate, for (a) a gas of classical atoms, (b) a gas of bosons, and (c) a gas of fermions. (15%)