

所別：太空科學研究所碩士班

科目：應用數學

1. Show that

(a)  $\hat{n} \cdot \vec{\nabla} \phi = \frac{\partial \phi}{\partial s}$  along the  $\hat{n}$  direction. (8%)

(b)  $\vec{\nabla} \times \vec{A} = 0$  is the sufficient and necessary condition of  $\vec{A} = \vec{\nabla} \phi$ . (12%)

2. Show that

(a) The eigenvalues of a real  $n \times n$  symmetrical matrix are real. (8%)

(b) Eigenvectors of a real  $n \times n$  symmetrical matrix belonging to different eigenvalues are orthogonal. (12%)

3. (a) Obtain the Fourier series representing  $f(x) = x$  in the interval  $(-L, L)$  with the period  $2L$ . (12%)

(b) Determine the value of series:  $s = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$  (8%)

4. Find the complete solution of the following differential equations.

(a)  $\frac{dy}{dx} + (y - 2 \sin x) \cos x = 0$ , (10%)

(b)  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2$ . (10%)

5. Use residue calculus to evaluate the following integrals:

(a)  $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx$ , (10%)

(b)  $\int_{-1}^1 \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ . (10%)

參考用