

國立中央大學八十六學年度碩士班研究生入學試題卷

所別： 應用地質研究所 不分組 科目：

微積分

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1) Evaluate 15%

a) $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$

b) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x)$

c) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left[\left(\frac{i}{n} \right)^2 + 1 \right]$



2) Find 20%

a) $\int (x+1)^2 \cos(x^3 + 3x^2 + 3x) dx$

b) $\int_{-1}^8 x^{-2/3} dx$

c) f , if $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$, and $f(1) = 1$

d) y' , if $y = x^{\sqrt{x}}$

3) Evaluate $\int_0^1 \frac{dx}{\sqrt{1+x^2}}$ and show that $\text{Sinh}^{-1} x = \ln(x + \sqrt{x^2 + 1})$ 10%

4) The half-life of radium-226 is 1590 years. A sample of radium-226 has a mass of 100 mg. Find (a) formula for the mass that remains after t years. (b) the mass after 1000 years correct to the nearest milligram. © when will the mass be reduced to 30 mg. 15%

5) Find the extreme values of the function $f(x,y) = x^2 + y$ on the circle $x^2 + y^2 = 1$. 10%

6) Solve 10%

a) $y'' - 4y = xe^x + \cos 2x$

b) $y'' + y = \tan x$, $0 < x < \pi/2$

7) The density at any point on a semicircular lamina is proportional to the distance from the center of the circle. Find the center of mass of the lamina. 10%

8) Evaluate $\oint_c y^2 dx + 3xy dy$, where c is the boundary of the semiannular region D in the upper half-plane between $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$. 10%