

國立中央大學98學年度碩士班考試入學試題卷

所別：水文與海洋科學研究所碩士班 一般生 科目：應用數學 共 1 頁 第 1 頁

\*請在試卷答案卷(卡)內作答

1. (a) [10%] Find the derivative with respect to  $x$  of  $f(t) = 2at$ , where  $x = at^3$ .  
(b) [10%] Find the derivative  $\frac{dy}{dx}$  if  $x^3 - 3xy + y^3 = 2$ .
2. [20%] Evaluate the integral  $\int e^{ax} \cos bx \, dx$ .
3. [20%] Find the Taylor series expansion up to quadratic terms in  $(x-2)$  and  $(y-3)$  of  $f(x, y) = ye^{xy}$  about the point  $x = 2, y = 3$ .
4. The vector field  $\mathbf{F}$  is defined by  $\mathbf{F} = 2xz\mathbf{i} + 2yz^2\mathbf{j} + (x^2 + 2y^2z - 1)\mathbf{k}$ .  
(a) [10%] Calculate  $\nabla \times \mathbf{F}$  and deduce that  $\mathbf{F}$  can be written as  $\mathbf{F} = \nabla \phi$ .  
(b) [10%] Determine the function  $\phi$ .
5. [20%] Solve the differential equation  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{-x}$  for  $y = y(x)$  subject to the boundary conditions:  $y(0) = 2, y'(0) = 1$ .

參考用