

# 國立中央大學八十五學年度碩士班研究生入學試題卷

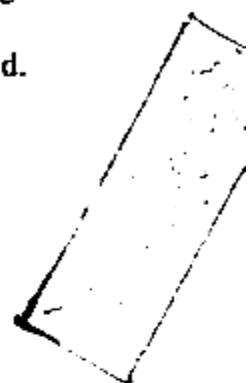
所別: 資訊管理研究所 丁組 科目: 離散數學 共    / 頁 第    / 頁

1. (10%)

(a). Write the adjacency matrix of each component of the graph G with the given adjacency matrix.

(b). Find the number of edge sequences of length 3 from vertex b to vertex d.

$$\begin{array}{c}
 a \\
 b \\
 c \\
 d \\
 e \\
 f \\
 g
 \end{array}
 \begin{array}{c}
 a \\
 b \\
 c \\
 d \\
 e \\
 f \\
 g
 \end{array}
 \begin{bmatrix}
 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
 1 & 0 & 1 & 0 & 0 & 0 & 1
 \end{bmatrix}$$



2. (10%) How many of the equivalence relations on  $A = \{a, b, c, d, e, f\}$  have (a) exactly two equivalence classes of size 3? (b) at least one equivalence class with three or more elements?

3. (15%) (a). If  $x$  is a bit string, let  $c(x)$  be the maximum number of consecutive 0's in  $x$ . (Examples:  $c(10010)=2$ ,  $c(00110001)=3$ .) Let  $S_n$  be the number of  $n$ -bit strings  $x$  with  $c(x) \leq 2$ . Develop a recurrence relation for  $S_n$ .

(b). Solve the recurrence relation

$$\sqrt{T_n} = \sqrt{T_{n-1}} + 2\sqrt{T_{n-2}},$$

with initial condition  $T_0 = T_1 = 1$ .

4. (15%) Determine whether each argument in (a), (b) and (c) is valid.

$$\begin{array}{ccc}
 \text{(a). } \frac{p \wedge \bar{p}}{\therefore q} & \text{(b). } \frac{p \rightarrow (q \rightarrow r) \quad q \rightarrow (p \rightarrow r)}{\therefore (p \vee q) \rightarrow r} & \text{(c). } \frac{(p \rightarrow q) \wedge (r \rightarrow s) \quad p \vee r}{\therefore q \vee s}
 \end{array}$$

5. (15%) Prove that the number of vertices ( $V$ ), edges ( $E$ ), and faces ( $F$ ) in an arbitrary connected planar graph are related by the formula  $V + F = E + 2$ .

6. (10%) Prove that  
It is possible to color the regions formed by any number of lines in the plane with only two colors.

7. (10%) Define what is NP-complete. Explain the important characteristics of NP-complete problems.

8. (15%) Use an example to explain Kruskal's algorithm for finding the minimal spanning tree, and prove that Kruskal's algorithm is correct.