# 國立中央大學八十五學年度碩士班研究生入學試題卷

所別：資訊管理研究所　丁組　科目：離散數學　共 / 頁 第 / 頁

1. (10%)
(a). Write the adjacency matrix of each component of the graph G with the given adjacency matrix.
(b). Find the number of edge sequences of length 3 from vertex b to vertex d.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

2. (10%) How many of the equivalence relations on A = {a,b,c,d,e,f } have (a) exactly two equivalence classes of size 3 ? (b) at least one equivalence class with three or more elements ?

3. (15%) (a). If x is a bit string, let c(x) be the maximum number of consecutive 0's in x. (Examples : c(10010)=2, c(00110001)=3.) Let $S_n$ be the number of n-bit strings x with c(x) ≤ 2. Develop a recurrence relation for $S_n$.
(b). Solve the recurrence relation

$$\sqrt{T_n} = \sqrt{T_{n-1}} + 2\sqrt{T_{n-2}},$$

with initial condition $T_0 = T_1 = 1$.

4. (15%) Determine whether each argument in (a),(b) and (c) is valid.

(a). $\dfrac{p \wedge \overline{p}}{\therefore q}$

(b). $\dfrac{p \rightarrow (q \rightarrow r) \quad q \rightarrow (p \rightarrow r)}{\therefore (p \vee q) \rightarrow r}$

(c) $\dfrac{(p \rightarrow q) \wedge (r \rightarrow s) \quad p \vee r}{\therefore q \vee s}$

5. (15%) Prove that the number of vertices ($V$), edges ($E$), and faces ($F$) in an arbitrary connected planar graph are related by the formula $V+F=E+2$.

6. (10%) Prove that
It is possible to color the regions formed by any number of lines in the plane with only two colors.

7. (10%) Define what is NP-complete. Explain the important characteristics of NP-complete problems.

8. (15%) Use an example to explain Kruskal's algorithm for finding the minimal spanning tree, and prove that Kruskal's algorithm is correct.