國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 資訊管理研究所 丁組 科目: 離散數學 共 1 頁 第 1 頁

本試卷共有十題，每題十分。

1. Suppose that R_i is a partial order on X_i, $i = 1, 2$. Show that R is a partial order on $X_1 \times X_2$ if we define
$$(x_1, x_2)R(x_1', x_2') \quad \text{if } x_1 R_1 x_1' \text{ and } x_2 R_2 x_2'.$$

2. Show that $3^n + 7^n - 2$ is divisible by 8, for $n = 1, 2, \ldots$.

3. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4\}$. Define the relation R on $\mathcal{P}(X)$, the set of all subsets of X, as
$$ARB \text{ if and only if } A \cup Y = B \cup Y.$$
   (a) Show that R is an equivalence relation.
   (b) How many distinct equivalence classes are there?

4. Assume that a robot can take steps of 1 meter, 2 meters, or 3 meters.
   (a) Write a recursive algorithm to calculate the number of ways the robot can walk n meters.
   (b) Give a proof using mathematical induction that your algorithm for part (a) is correct.

5. Let $O(\cdot)$ be the big-oh notation. Show that x^2 is not $O(x)$.

6. Calculate the number of relations on a set of n elements with each of the following properties respectively.
   (a) Symmetric
   (b) Asymmetric
   (c) Antisymmetric
   (d) Reflexive
   (e) Reflexive and symmetric

7. Calculate the number of equivalence relations with exactly 3 equivalence classes on a set with 5 elements.

8. Find the maximum number of edges in a simple disconnected graph with n vertices.

9. For each of the following statements, provide an example if it is possible or prove that it is impossible.
   (a) In a group of 5 persons, each of them has to shake hands with exactly 2 other persons in the group.
   (b) In a group of 7 persons, each of them has to shake hands with exactly 3 other persons in the group.

10. Draw all the spanning trees (labeled, unrooted) of the following graphs.
   (a) K_3, the complete graph with 3 vertices.
   (b) $K_{2,2}$, the complete bipartite graph with 2 sets of 2 vertices.