

所別：資訊管理學系碩士班 丙組 科目：離散數學

1. (10%) Determine the number of positive integers n where $n \leq 10000$ and n is not divisible by 2, 3 or 5.
2. (10%) What is the coefficient of $x^2y^3z^2w^5$ in the expression of $(x+2y-3z+2d+5)^{16}$?
3. (10%) Solve the recurrence relation $a_n - 3a_{n-1} = 5(3^n)$ for $n \geq 1$ and $a_0 = 2$.
4. (20%) Show that $\sqrt{2}$ is irrational.
5. John 說了下面兩句話，這兩句話要就都是實話，要就都是謊話。試問 John 是否愛 Lucy? John 是否愛 Vivian? 請說明你的理由。
I love Lucy.
I love Lucy if and only if I also love Vivian. (10%)
6. 試證明：一個 $2^N * 2^N$ 的棋盤，任意缺掉一塊以後，一定可以用一個「三個方塊所構成的 L 型」給填滿。8*8 的參考圖形如圖 1。 (10%)

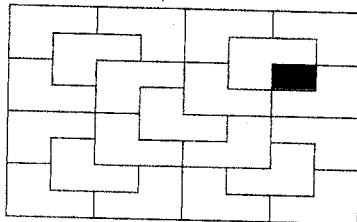


圖 1

7. Let R be a binary relation on the set of all positive integers such that $R = \{(a, b) \mid a - b \text{ is an odd positive integer}\}$
Is R reflexive? Symmetric? Antisymmetric? Transitive? An equivalence relation? A partial ordering relation? Why? (15%)
8. 試証在 6 個人中，一定有三個人彼此是朋友，要不然就一定有三個人彼此不認識。(提示：利用 Pigeonhole principle) (15%)

參考用