

考生請注意

請依序作答，未作答題，書寫題號後留空白，違者扣總分 10 分

- (10%) Suppose that a person plays a game in which his score must be one of the 50 numbers 1,2,3,...,50 and that each of these 50 numbers is equally likely to be his score. What is the probability that the person gets the score that will be 50.
- (10%) Let X be a continuous random variable where $f(x) = cx^2$; $0 < x < 1$, Find the value of c .
- (10%) A bottling machine can be regulated so that it discharges an average of μ ounces per bottle. It has been observed that the amount of fill dispensed by the machine is normally distributed with $\sigma = 1.0$ ounce. How many observations should be included in the sample if we wish the ounces of fill to be within 0.3 ounces of μ with probability 0.95?
- (10%) A random sample of 20 boys and 15 girls were given a standardized test. The average grade of the boys was 78 with a standard deviation of 6, while the girls made an average grade of 84 with a standard deviation of 8. Test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against the alternate hypotheses $\sigma_1^2 < \sigma_2^2$ where σ_1^2 and σ_2^2 are the variances of the population of boys and girls. Use a 0.05 level of significance.
(Note: $F_{0.95}(15,20) = 2.33$, $F_{0.95}(14,19) = 2.36$, $F_{0.95}(14,20) = 2.39$)
- (10%) A vice-president in charge of sales for a large corporation claims that salesmen are averaging no more than 15 sales contacts per week. As check on the claim, $n = 36$ salesmen are selected at random, and the number of contacts is recorded for a single randomly selected week. The sample reveals a mean of 17 contacts and a variance of 9. Does the evidence contradict the vice-president's claim? (Use $\alpha = 0.05$, $Z_\alpha = 1.645$)
- (10%) Following the question (5), suppose that the vice-president wants to be able to detect a difference equal to one call in the mean number of customer calls per week. That is, the vice-president is interested in testing $H_0: \mu = 15$ against $H_a: \mu = 16$. Find the type-II-error probabilities.
(Note: $Z_{-0.26} = 0.3974$, $Z_{-0.30} = 0.3821$, $Z_{-0.36} = 0.3594$, $Z_{-0.40} = 0.3446$)



7. (20%) The number of accidents per week, Y , at a certain intersection was checked for $n=50$ weeks, with the results as shown in the following table:

| Y | Frequency |
|-----------|-----------|
| 0 | 32 |
| 1 | 12 |
| 2 | 6 |
| 3 or more | 0 |

Test the hypothesis that the random variable Y has a Poisson distribution, assuming the observations to be independent. Use $\alpha = 0.05$.

Note: $\chi^2_{0.05}(1) = 3.841$

| λ | $e^{-\lambda}$ |
|-----------|----------------|
| 0.28 | 0.756 |
| 0.38 | 0.684 |
| 0.48 | 0.619 |
| 0.58 | 0.600 |

8. (20%) A famous astronomer observes the number of sunspots and applications to medical school for 5 successive years. It is his theory that somehow the number of sunspots is responsible for the number of applications to medical school. Find the correlation between these two quantities. How well is the variation in applications explained by the variation in the number of sunspots? If no sunspots are observed in a year, how many applicants to medical school are predicted?

The observations are:

| X (observed sunspots) | Y (applicants in thousands) |
|--------------------------|--------------------------------|
| 3 | 9 |
| 1 | 5 |
| 2 | 7 |
| 5 | 14 |
| 4 | 10 |