所別：產業經濟研究所碩士班 甲組 科目：微積分

可選擇以英文或中文作答。

1. (20%) Suppose the value of timber (already planted on some given land) is a function of time, $V = 2^{\sqrt{t}}$. Assume that the discount rate is r and that there is no upkeep cost during the period of timber growth. What is the optimal time to cut the timber for sale?

2. (20%) What is implicit function theorem. Provide an example for the application of implicit function theorem.

3. (20%) Solve the differential equation $\frac{dy}{dt} + Ry = Ty^m$, where R and T are two functions of t, and m is any number other than 0 and 1.

4. (40%) Consider the CES production function, $Q = A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho}$, where $A > 0$, $0 < \delta < 1$, $-1 < \rho \neq 0$.

   (a) Show that, on an isoquant of the CES production function, $\frac{d^2K}{dL^2} > 0$.

   (b) Show that the function satisfies Euler's theorem (i.e., $K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = Q$).