

注意事項：

- 請列出計算過程，僅有答案，不予計分。
- 請務必將題號標示清楚。可不按順序作答。

1. Find the following integrals. (20%)

$$(a) \int \frac{\sqrt{x}}{\sqrt{x^3-1}} dx$$

$$(b) \int_3^{\infty} \frac{\ln x}{\sqrt{x^2-2}} dx$$

$$(c) \int \arcsin(2x) dx$$

$$(d) \int_0^{\ln 2} \frac{e^x}{e^{2x} + 3e^x + 2} dx$$

2. Find the following limits. (20%)

$$(a) \lim_{n \rightarrow \infty} \left( \frac{n+6}{n-1} \right)^n$$

$$(b) \lim_{x \rightarrow \infty} (\sin \sqrt{x+2} - \sin \sqrt{x})$$

$$(c) \lim_{x \rightarrow \infty} \frac{x \cos x}{x+1-\cos x}$$

$$(d) \lim_{x \rightarrow \infty} x[\ln(x+2) - \ln x]$$

3. Determine the convergence or divergence of the following series. (24%)

$$(a) \sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n(1+\ln n)}$$

$$(b) \sum_{n=1}^{\infty} \sqrt{n} \left( \frac{2}{3} \right)^n$$

$$(c) \sum_{n=1}^{\infty} (-1)^n \frac{\ln(2^n)}{n}$$

$$(d) \sum_{n=1}^{\infty} (\ln \sqrt{n+1} - \ln \sqrt{n})$$

4. Let  $f(x) = \frac{x}{(1-x)^2}$ .

(a) Find a power series of  $f(x)$ , centered at 0, and determine its interval of convergence. (8%)

(b) Evaluate  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}$ . (8%)

5. Let  $m, n$  be positive integers. Define

$$B(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx.$$

(a) Show that  $B(m, n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}$ . (8%)

(b) Show that  $B(m, n) = 2 \int_0^{\pi/2} (\sin \theta)^{2m-1} (\cos \theta)^{2n-1} d\theta$ . (7%)

(c) Evaluate  $\int_0^{\pi/2} (\sin \theta)^9 (\cos \theta)^{13} d\theta$ . (5%)