所別：工業管理研究所碩士班 不分組(一般生)　科目：統計學　共 1 頁　第 1 頁

本科考試可使用計算器，廠牌、功能不拘　＊請在試卷答案卷（卡）內作答

參考用

1. A load of laundry contains 4 (2 pairs) tan socks and 4 grey socks. A color-blind person who cannot see the difference between tan and grey sorts the socks into pairs and gets 4 pairs of mismatched socks. Assume that pairing of socks is random. Let X be the number of mismatched pairs. Find the probability mass function of X. (15 points)

2. The moment generating function of random variable X is defined as follows:

$$M_X(t) = E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} f(x)dx$$

, where $f(x)$ is the pdf of X.

(a) Find the moment generating function of X, where X is $U(0,1)$ distributed. (5 points)

(b) Find $E[Y^2]$ from the moment generating function of Y, where $Y = X + Z$, X and Z are independent and $U(0,1)$ distributed. (10 points)

3. Consider mutually independent $U(0,1)$ random variables $X_1, X_2, \ldots$. We are interested in the pdf of Z_n, $f_n(t)$, where $Z_n = X_1 X_2 \ldots X_n$. Let $F_n(t) = P(Z_n \le t)$, where $0 < t < 1$. By definition, $F_1(t) = t$ and note that $Z_n = Z_{n-1} X_n$.

(a) Suppose that F_{n-1} is known. What is $P(Z_n \le t | X_n = x)$ in terms of F_{n-1} ? (10 points)

(b) From (a), what is $F_n(t)$ for $0 \le t \le 1$ in terms of F_{n-1}? (5 points)

(c) From (b), what is $F_3(t)$? (5 points)

4. Assume that there is a relationship between random variable Y_i and the corresponding given value X_i as following:

$$Y_i = \beta X_i + \varepsilon_i \qquad i = 1,2,\ldots,n$$

where β is an unknown parameter, and ε_i is independently and normally distributed with mean 0, and variance σ^2.

(a) Find the least squares estimator $\hat{\beta}$, such that $\sum_{i=1}^{n}[Y_i - \hat{\beta} X_i]^2$ minimized. (10 points)

(b) Show that whether or not the least squares estimator $\hat{\beta}$ in (a) is the same as the maximum likelihood estimator. (5 points)

(c) Given a particular value, x_p, what is the distribution of y_p , where $y_p = \beta x_p + \varepsilon_i$. (5 points)

(d) Given a particular value, x_p, what is the distribution of $\hat{y}_p$, where $\hat{y}_p = \hat{\beta} x_p$. (5 points)

(e) Construct an 100(1-α)% confidence interval of the expected value of y_p, $E(y_p)$. (5 points)

5. A supplier ships parts to another company in lots of *N* parts, some of which could be defective. The receiving company uses an acceptance sampling plan (single-sampling plan) which defined by the sample size *n* without replacement and the acceptance number *c*. i.e. from a lot of size *N*, a random sample of *n* parts is inspected. If the number of defective items is less than or equal to *c*, the lot will be accepted.

(a) For a particular lot, assume the defective rate is *p*. Under the sampling plan (*n*=5, *c*=1), please find the probability of accepting the lot. (5 points)

(b) In (a), the accepting probability depends on the defective rate, *p*. If the acceptable quality level (AQL) is set at *p*=0.1, and the rejectable quality level (RQL) is *p*=0.2, the sampling plan is designed for the test H_0: *p*=0.1 v.s. H_A: *p*=0.2. Please find the type I error, α, and the type II error, β, under the sampling plan (*n*=5, *c*=0). (5 points)

(c) If the desired type I error is α=0.05, by using the sampling plan (*n*, *c*=0), what is the minimal required sampling size, *n*? What is the corresponding type II error, β? (5 points)

(d) Furthermore, what is your suggested sampling plan to achieve α and β are both less than 0.05 simultaneously. (5 points)