# 國立中央大學 105 學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生) 共6頁 第1頁

科目： 統計學

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**Part I Multiple-Choice Questions:**

**Identify the letter of the choice that best completes the statement or answers the question. There are 20 multiple-choice questions. 2.5 points for each question.**

1. A government report gives a 90% confidence interval for the 2014 median household monthly income, $\mu$, as \$45,816 ± \$314. The survey was based on a random sample of about 5,000 households. If the null hypothesis is stated as $H_0$: $\mu = 45{,}000$, which following conclusion can be made?

a. Do not reject $H_0$ using $\alpha = 0.05$;
b. Do not reject $H_0$ using $\alpha = 0.10$;
c. Reject $H_0$ using $\alpha = 0.10$;
d. $p$-value is greater than 0.2.

2. It is known that the mean of a population is 88. A sample of size 50 is taken and the sample mean is computed as 85. You decided to reject the null hypothesis that $\mu \geq 90$.

a. you have committed a Type I error;
b. you have committed a Type II error;
c. you have committed either Type I or Type II error;
d. you have neither committed Type I nor Type II error.

3. In a two-tailed hypothesis test, $z$-statistic is computed as $z = -2.000$. The $p$-value is:

a. $2 \times \text{Prob}(Z > 2)$;
b. $2 \times \text{Prob}(|Z| > 2)$;
c. $\text{Prob}(Z < -2)$;
d. -2.000.

4. If the alternate hypothesis is $\mu > 15$, one would tend to reject null hypothesis if

a. the sample mean is significantly greater than 15;
b. the sample mean is significantly smaller than 15;
c. the sample mean is equal to 15;
d. the sample mean is 15.01.

5. Suppose $p$-value is equal to 0.026. The null hypothesis can be rejected

a. if $\alpha = .005$;
b. if $\alpha = .01$;
c. if $\alpha = .05$;
d. not enough data to make a decision if the null hypothesis can be rejected.

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6. In what kind of following situations can we make a conclusion that the data are significant to reject null hypothesis in two-tailed test?

a. $p$-value > $\alpha$
b. $p$-value < $\alpha$
c. $p$-value > $2\alpha$
d. $p$-value < $2\alpha$

7. Which of the following statements is NOT true about the level of significance in a test of hypothesis?

a. The larger the level of significance, the more likely you are to reject the null hypothesis;
b. The level of significance is the maximum risk we are willing to accept in making a Type I error;
c. The level of significance is also known as the alpha level;
d. The level of significance is another name for the $p$-value.

8. A random sample of 100 people was taken. 55 of them favored Candidate A. At 95% confidence, it can be concluded that the proportion of the population in favor of candidate A

a. is significantly smaller than 55%;
b. is significantly greater than 55%;
c. is significantly greater than 50%;
d. is not significantly greater than 50%.

9. To construct an interval estimate for the difference between the means of two populations which are normally distributed and have equal variances, we must use a t distribution with (let $n_1$ be the size of sample 1 and $n_2$ the size of sample 2)

a. $(n_1 + n_2)$ degrees of freedom;
b. $(n_1 + n_2 - 1)$ degrees of freedom;
c. $(n_1 + n_2 - 2)$ degrees of freedom;
d. $(n_1 - n_2 + 2)$ degrees of freedom.

10. Which following statement is NOT right for sample mean $\bar{x}_1 - \bar{x}_2$?

a. $\bar{x}_1 - \bar{x}_2$ is a parameter because $\bar{x}_1$ and $\bar{x}_2$ are population parameters;
b. $\bar{x}_1 - \bar{x}_2$ is an estimate for $\mu_1 - \mu_2$;
c. $\bar{x}_1 - \bar{x}_2$ is always the middle point of the confident interval for $\mu_1 - \mu_2$;
d. $\bar{x}_1 - \bar{x}_2$ is a random variable with expect value $\mu_1 - \mu_2$.

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11. In testing $H_0$: $\mu_1 = \mu_2$ against $H_a$: $\mu_1 \neq \mu_2$, we obtained the value of the z-test statistic to be 1.35. Given that Prob($z$>1.35)=0.0885. We can conclude that

a. there is a significant difference between the two population means at $\alpha$=5% since the p-value = 0.0885 > 0.05;

b. there is a significant difference between the two population means at $\alpha$=5% because the p-value = 0.177 > 0.05;

c. there is a significant difference between the two population means at $\alpha$=10%, since the p-value = 0.0885 < 0.1;

d. there is no significant difference between the two population means at $\alpha$=10%, since the p-value = 0.177 > 0.1.

Answer questions 12-14 based on the following ANOVA table.

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | F |
|---|---|---|---|---|
| Treatments | 64 | | | 8 |
| Error | | | 2 | |
| Total | 184 | | | |

12. How many populations (treatments) are there corresponding to this ANOVA table?

a. 3;

b. 4;

c. 5;

d. Cannot be determined

13. How many observations are there corresponding to this ANOVA table?

a. 65;

b. 66;

c. 185;

d. 186.

14. Given that $F_{0.05,4,60} = 2.53$, $F_{0.05,60,4} = 5.69$, $F_{0.025,4,60} = 3.01$, $F_{0.025,60,4} = 8.36$, $F_{0.01,4,60} = 4.13$, and $F_{0.01,60,4} = 13.65$, if we want to determine whether or not the means of the populations are equal, the conclusion of the test could be

a. the means are not equal at $\alpha = 0.01$;

b. the means are not equal at $\alpha = 0.025$;

c. the means are not equal at $\alpha = 0.05$;

d. all of the above are correct.

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15. 10 students are randomly selected from a statistics class and the midterm and the final exam scores of these students in the statistics course are tabulated below. What are the appropriate null and alternative hypotheses if we want to test to determine whether the final exam scores are better than the midterm exam scores?

| Midterm | 70 | 74 | 80 | 84 | 80 | 67 | 70 | 64 | 74 | 82 |
|---|---|---|---|---|---|---|---|---|---|---|
| Final | 87 | 79 | 88 | 98 | 96 | 73 | 83 | 79 | 91 | 94 |

a. $H_0: \mu_d = 0, H_a: \mu_d \neq 0$;
b. $H_0: \mu_d \leq 0, H_a: \mu_d > 0$;
c. $H_0: \mu_1 - \mu_2 \geq 0, H_a: \mu_1 - \mu_2 < 0$;
d. $H_0: \mu_1 - \mu_2 \leq 0, H_a: \mu_1 - \mu_2 > 0$.

16. Independent simple random samples are taken to test the difference between the means of two populations whose standard deviations are not known. The sample sizes are $n_1 = 25$ and $n_2 = 35$. The correct distribution to use is the

a. Normal distribution;
b. $t$ distribution with 60 degrees of freedom;
c. $t$ distribution with 59 degrees of freedom;
d. $t$ distribution with 58 degrees of freedom.

17. The manager of a grocery store wants to determine what proportion of people who enter his store are his regular customers. What is the minimum sample size he should take so that at 98% confidence the margin of error will not be more than 0.05? Given that $z_{0.05} = 1.645$, $z_{0.04} = 1.751$, $z_{0.03} = 1.881$, $z_{0.02} = 2.054$, $z_{0.01} = 2.326$;

a. 30;
b. 385;
c. 421;
d. 543.

18. $X$ is a normal random variable with mean $\mu$ and standard deviation $\sigma$. Regarding the sample mean ($\overline{X}$), which one of the following statements is *true*?

a. As the sample size increases, the standard deviation of $\overline{X}$ increases;
b. As the sample size increases, the expected value of the sample mean remains the same;
c. As the sample size increases, the $100(1-\alpha)\%$ confidence interval becomes wider;
d. As the sample size increases, $P(\overline{X} > \mu)$ increases.

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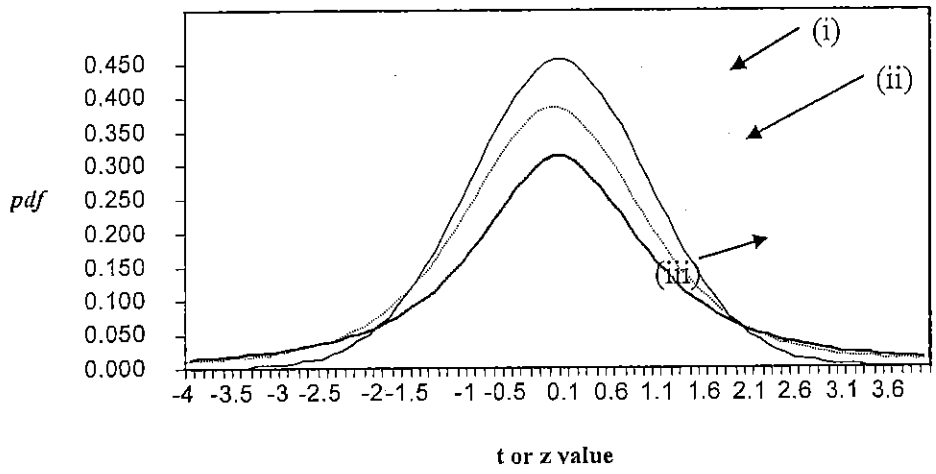
19. A standard normal distribution and two t-distribution curves are drawn below. Of the t-distribution curves, which one has the higher degrees of freedom?




a. (i);
b. (ii);
c. (iii);
d. Insufficient evidence to make a conclusion

20. A sample of size 22 is taken from a normally distributed population with a mean of 10 and a standard deviation of 3. The probability that the sample mean is equal to 10 is

a. 0;
b. 0.5;
c. -0.5;
d. 1

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**Part II**

1. Consider the following continuous random vector $(X_1, X_2)$ with density

$$f(x_1, x_2) = 4e^{-2(x_1+x_2)},$$

$x_1 > 0$ and $x_2 > 0$. We are interested in the distribution of $Y = H_1(X_1, X_2) = X_1/X_2$.

(a) Introduce a second random variable $Z = H_2(X_1, X_2)$. The function of $H_2$ is selected for convenience so that we can solve $y = H_1(x_1, x_2)$ and $z = H_2(x_1, x_2)$ for $x_1$ and $x_2$ in terms of $y$ and $z$. What is function $H_2$? (8 points)
(Hint: The function you choose will have a huge impact on the derivation for the rest of the problem. Be Wise!)

(b) Determine $x_1$ and $x_2$ in terms of $y$ and $z$ (5 points)

(c) Determine the joint pdf for $(Y, Z)$ (7 points)

(d) Determine the marginal pdf for $Y$ (10 points)

2. Let $X_1$ and $X_2$ be independent uniformly distributed over the integers 0, 1, 2, ... m. Define the distance between $X_1$ and $X_2$ as

$$D = |X_1 - X_2|.$$

(a) Determine the pmf for $D$. (10 points)

(b) Show that

$$E[D] = \frac{1}{3}m + \frac{1}{3}\left(\frac{m}{m+1}\right)$$

(10 points)

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