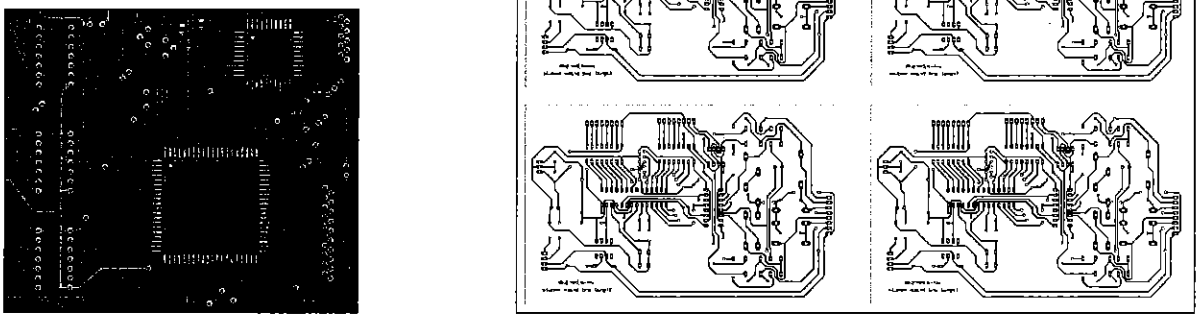
國立中央大學 105 學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生)　　共 3 頁 第 1 頁

科目： 作業研究

本科考試禁用計算器　　＊請在答案卷(卡)內作答

1. (40 分). A printed circuit board (PCB) is a "non-conductive substrate" used to mechanically support a variety of electronic components (such as CPUs, memories, transistors) and electrically connect these components. PCBs are normally manufactured in groups; in other words, a certain number of PCBs will go through manufacturing processes together until all required operations are done. In the two figures shown below, the left one shows what a PCB may look like and the right one is a sheet of 4 identical PCBs (therefore, they will go through every manufacturing operation together).



Figures: a PCB (left); a sheet of 4 identical PCBs with holes to be drilled (right).

A critical operation for making a PCB is "hole drilling." As shown above in the right figure, each PCB has a number of holes to be drilled. Normally, a laser is used for drilling these holes and it will drill *one hole at a time*. To be more precise, the laser will move to the top of a location on the PCB where a hole is needed and then fire a laser beam to drill the hole. Then, the laser will move to another location on the PCB where a hole is needed there and fire the laser beam again to drill the hole. The laser will repeat this move-and-fire-laser process until it has drilled all the holes required for that PCB; then, it will move to another PCB and start the move-and-fire-laser process all over again.

The above actually describes an interesting optimization problem: First of all, each sheet of PCBs has a certain number of holes to be drilled and their locations on the PCB are given. Next, the laser moves between the locations of any two holes in straight line and at constant speed. Since it takes time to move the laser, we naturally want the drilling process to finish *as quickly as possible* for each sheet of PCBs.

注意：背面有試題

所別： 工業管理研究所碩士班 不分組(一般生)　　共3頁 第2頁

科目： 作業研究

本科考試禁用計算器　　＊請在答案卷(卡)內作答

- (10 分) What must be minimized for the laser so it can drill all the holes as quickly as possible?
- (10 分) To formulate this problem as a linear mixed-integer programming (LMIP) model, what are the key decision variables? Please define just one decision variable that, in your opinion, is the key decision to solving the problem.
- (10 分) Let d_{ij} represent the distance for the laser to move from location i to location j on the sheet of PCBs. Then what is the objective function for your LMIP model?
- (10 分) Finally, what are the constraints for your LMIP model for solving this problem?

2. (10 分) Please use the simplex method to solve the following problem and report all optimal solutions (if they exist).

$$\begin{aligned} \text{Minimize} \quad & -4x_1 - 4x_2 \\ \text{Subject to} \quad & x_1 - x_2 - x_3 = 1 \\ & -3x_1 + 3x_2 + 6x_3 - 3x_4 = 3 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

注意:背面有試題

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3. (5 * 6 = 30 points)

The computer lab has a help desk to assist students, who are served based on a first-come, first-served priority rule. On average, 15 students per hour arrive and arrivals are described using a Poisson distribution. The help desk helps an average of 20 students per hour, with the service rate being described by an exponential distribution. Calculate the following operating characteristics of the service system.

(A) The average utilization of the help desk server　　$P = \frac{\lambda}{\mu}$

(B) The average number of students in the system　　$L = \frac{\lambda}{\mu - \lambda}$

(C) The average number of students waiting in line　　$L_Q = pL$

(D) The average time a student spends in the system　　$W = \frac{1}{\mu - \lambda}$

(E) The average time a student spends waiting in line　　$W_Q = pW$

(F) The probability of having more than 4 students in the system　　$P_n = (1 - p)p^n$

4. (10 * 2 = 20 points)

A supermarket stocks three brands of coffee – A, B and C – and customers switch from brand to brand.

- The probability that a customer who has last bought A will buy A next time is 0.70; for B is 0.20；for C is 0.10.
- The probability that a customer who has last bought B will buy A next time is 0.35; for B is 0.50；for C is 0.15.
- The probability that a customer who has last bought C will buy A next time is 0.25; for B is 0.30；for C is 0.45.

(1) Develop the transition matrix

(2) Estimate the long-term purchasing tendency (steady-state probabilities) of customers.

(Hint: $\pi = \pi \mathbf{P}$, $\Sigma \pi_i = 1$)

Brand A will eventually have a _____% share of customers, brand B a _____% share, and brand C a _____% share.