國立中央大學九十一學年度碩士班研究生入學試題卷

1. Please answer the following problems:

   (a) (5%)Give an example table of simplex method, from which we can conclude that the linear programming problem we want to solve is unbounded.(Our objective is to maximize the objection function)

   (b)(5%)Give an example final table of simplex method, from which we can conclude that there are infinite many optimal solutions for the linear programming problem we want to solve.(Our objective is to maximize the objection function)

   (c)(5%)What are the results of a linear programming problem that can be found by using simplex method? When do we need to use big-M simplex method to solve a linear programming problem.

   (d)(5%)Give a real world example that we can model it as a minimum spanning tree problem to find the solution.

   (e)(5%)Give a real world example that we can model it as a maximum flow problem to find the solution.

2. Consider the following linear programming problem

$$\begin{aligned} \max Z = 4x_1 + 2x_2 + 3x_3 + 5x_4 & \\ \text{s.t.} 8x_1 + x_2 + x_3 + 5x_4 &\leq 300 \\ 2x_1 + 3x_2 + 4x_3 + 2x_4 &\leq 300 \\ 2x_1 + x_2 + 2x_3 + x_4 &\leq 400 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. & \end{aligned}$$

   (a) (10%)Use simplex method to find the optimal solution if it exists. Please also identify the basic matrix and its inverse at each step.

   (b) (5%)Write down the dual problem of this problem and write down the optimal solution of this dual problem.

   (c) (10%)Now consider the following linear programming problem

$$\begin{aligned} \max Z = 4x_1 + 2x_2 + 3x_3 + 5x_4 & \\ \text{s.t.} 8x_1 + x_2 + x_3 + 5x_4 &\leq 300 \\ 2x_1 + 3x_2 + 4x_3 + 2x_4 &\leq 300 \\ 2x_1 + x_2 + 2x_3 + x_4 &= 400 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. & \end{aligned}$$

   Use two-phase method to check if it is feasible and find the optimal solution if it is feasible.

3. A job shop consists of three machines and two repairmen. The amount of time a machine works before breaking down is exponentially distributed with mean λ. The broken machines are fixed on the first-come-first-served basis. If the amount it takes a single repairman to fix a machine is exponentially distributed with mean μ.

   (a)(5%) Describe this system as a continuous time Markov chain. Is it a birth and death process?

   (b) (5%)What is the average number of machines not in use? What is the utilization of any machine?

   (c)(5%) What proportion of time are both repairmen busy? What is the utilization of any repairman?

4. Consider a M/M/2/4 queueing system with arrival rate 1 per hour and service rate 2 per hour.

   (a) (5%)Find the limiting probabilities.

   (b) (5%)What is the blocking probability? What is the effective arrival rate?

   (c) (5%)What is the average number of busy servers? What is the utilization of any server?

   (d) (5%)What is the average number of customers in system?

   (e) (5%)What is the average waiting time in system of a customer(included those rejected customers, whose waiting times are zero)? What is the average waiting time in system of an entering customer?

   (f) (5%)What is the probability that an entering customer must wait?

   (g) (5%)Consider the a M/M/2 system with same arrival rate and service rate. Find that probability that the system is empty.