國立中央大學98學年度碩士班考試入學試題卷

所別:工業管理研究所碩士班 甲組 科目:微積分 共 / 頁 第 / 頁

*請在試卷答案卷(卡)內作答

- 1. (50 points) Consider a manufacturer who produces and sells a certain product in the market with uncertain demand. The product is subject to a fixed life-time (such as newspapers, fashion goods, and agricultural produces). Let
 - c = manufacturing cost per unit;
 - s = salvage value per unit:
 - g = goodwill cost per unit due to stock-out;
 - p = selling price per unit;
 - Q = amount produced by the manufacturer;
 - f(x) = probability density function of demand; and

$$F(k) = \int_0^k f(x)dx$$
, the cumulative frequency distribution of demand.

The following relationship on the values is assumed to hold:

$$s < c < p$$
.

If the manufacturer produces quantity Q and sells to the customer, his expected profit will be given by

$$EP(Q) = -cQ + \int_0^Q [xp + s(Q - x)]f(x)dx + \int_Q^\infty [pQ - g(x - Q)]f(x)dx$$

(a) (10%) Explain, in detail, the managerial implications for the second term of EP(Q):

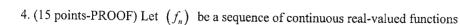
$$\int_0^Q [xp + s(Q - x)] f(x) dx$$

(b) (10%) Explain, in detail, the managerial implications for the third term of EP(Q):

$$\int_{Q}^{\infty} [pQ - g(x - Q)] f(x) dx.$$

- (c) (10%) Find the first order necessary condition (FONC) of EP(Q) (i.e., differentiating EP(Q) with respect to Q and setting this amount equal to zero).
- (d) (10%) Find the optimal production quantity Q^* such that the manufacturer's expected profit is maximized (i.e., solving the FONC).
- (e) (10%) Prove that the second order sufficient condition (SOSC) holds
- 2. (20 points-PROOF) Let $1 < a \le \sqrt{2}$. Let $x_0 = a$, and for $n \ge 1$, $x_n = a^{x_{n-1}}$.
- (a) (10 points) Show that $x_n < 2, n = 1, 2, \dots$
- (b) (10 points) Show that $\lim_{n\to\infty} x_n$ exists
- 3. (15 points-PROOF) Let $f(x) \ge 10$, f is continuous on [0,1]. Suppose that

$$\int_{0}^{1} f(x)dx = 10. \text{ Find all such } f.$$



on [0,1] defined by
$$f_n(x) = \frac{(-1)^{3n}}{3^n} \sin(2\pi nx^2), x \in [0,1], \forall n = 1, 2, \dots$$
 Show that

sequence
$$(g_n)$$
 defined by $g_n = \sum_{k=1}^n f_k$ converges.

