國立中央大學98學年度碩士班考試入學試題卷

所別：工業管理研究所碩士班 甲組　　科目：微積分　　共 1 頁　第 1 頁

＊請在試卷答案卷（卡）內作答

1. (50 points) Consider a manufacturer who produces and sells a certain product in the market with uncertain demand. The product is subject to a fixed life-time (such as newspapers, fashion goods, and agricultural produces). Let
   c = manufacturing cost per unit;
   s = salvage value per unit;
   g = goodwill cost per unit due to stock-out;
   p = selling price per unit;
   Q = amount produced by the manufacturer;
   $f(x)$ = probability density function of demand; and
   $F(k)=\int_0^k f(x)dx$, the cumulative frequency distribution of demand.

   The following relationship on the values is assumed to hold:
   $$s<c<p.$$
   If the manufacturer produces quantity Q and sells to the customer, his expected profit will be given by
   $$EP(Q)=-cQ+\int_0^Q [xp+s(Q-x)]f(x)dx+\int_Q^\infty [pQ-g(x-Q)]f(x)dx.$$

   (a) (10%) Explain, in detail, the managerial implications for the second term of $EP(Q)$:
   $$\int_0^Q [xp+s(Q-x)]f(x)dx.$$
   (b) (10%) Explain, in detail, the managerial implications for the third term of $EP(Q)$:
   $$\int_Q^\infty [pQ-g(x-Q)]f(x)dx.$$
   (c) (10%) Find the first order necessary condition (FONC) of $EP(Q)$ (i.e., differentiating $EP(Q)$ with respect to Q and setting this amount equal to zero).
   (d) (10%) Find the optimal production quantity Q^* such that the manufacturer's expected profit is maximized (i.e., solving the FONC).
   (e) (10%) Prove that the second order sufficient condition (SOSC) holds.

2. (20 points-PROOF) Let $1<a\le\sqrt{2}$. Let $x_0=a$, and for $n\ge 1$, $x_n=a^{x_{n-1}}$.
   (a) (10 points)Show that $x_n<2, n=1,2,\cdots$.
   (b) (10 points) Show that $\lim_{n\to\infty} x_n$ exists

參考用

3. (15 points-PROOF) Let $f(x)\ge 10$, f is continuous on $[0,1]$. Suppose that $\int_0^1 f(x)dx=10$. Find all such f.

4. (15 points-PROOF) Let (f_n) be a sequence of continuous real-valued functions on $[0,1]$ defined by $f_n(x)=\frac{(-1)^{3n}}{3^n}\sin(2\pi nx^2), x\in[0,1], \forall n=1,2,\cdots$. Show that sequence (g_n) defined by $g_n=\sum_{k=1}^{n} f_k$ converges.