國立中央大學98學年度碩士班考試入學試題卷

所別：工業管理研究所碩士班 乙組　科目：作業研究　共 3 頁　第 1 頁

＊請在試卷答案卷（卡）內作答

1. (15 分) A set C in the n-dimensional Euclidean space (n 維度歐基里德空間) is called a convex set if C satisfies the following condition:

   For any two points in C, say, $x_1 \in C$ and $x_2 \in C$, and for any $\lambda \in [0,1]$, we always have $\lambda x_1 + (1-\lambda)x_2 \in C$.

   Now, let C be this set: $\{x : Ax = b\}$, where A is an $m \times n$ matrix (both m, n are integers which are greater than or equal to 2), x is an $n \times 1$ vector, and b is an $m \times 1$ vector. Please verify if C is a convex set or not (請驗證 C 是不是一個 convex set).

2. (15 分) Given a convex set (refer to question 1 for the definition of a convex set) C in the 2-dimensional Euclidean space, vector $d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$ is called a direction of C if the following condition holds:

   For any point $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in C and any non-negative real number $k \geq 0$,

   the new point $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + k\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$ is still in C.

   Now, let C be the following convex set:

   $2x_1 + x_2 \leq 100$

   $x_1 + x_2 \leq 80$

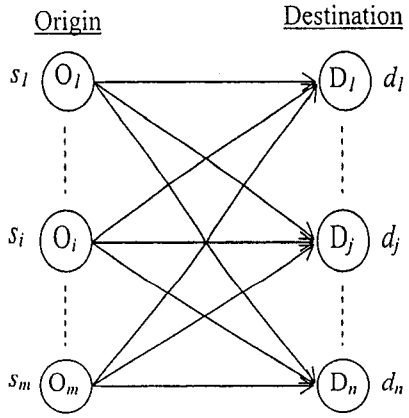
   $x_1 \leq 40$

   $x_1, x_2 \geq 0$

   Do you think if C has any direction? (請問 C 能否有任何的 direction？) Please answer "Yes – C has directions," "No – C cannot have any directions," or "There is not enough information to determine whether or not C has directions." Please also write down the reason to support your answer.

3.（20 分）

Consider the following transportation (運輸) problem:



There are m origin points, where origin i (O_i) has a supply of s_i ($s_i > 0$) iPhones. There are also n destination points, where destination j (D_j) requires d_j ($d_j > 0$) iPhone; that is, d_j is the demand at D_j. Associated with each link (O_i, D_j), there is a unit cost c_{ij} for transportation; that is, the cost of transporting each iPhone from O_i to D_j is c_{ij}. Also, for each unit of unsatisfied demand at D_j, the penalty cost is p_j. The problem is to find a feasible way to transport iPhones from origin points to destination points that minimizes the total cost (transportation and possible penalty).

Let x_{ij} be the number of iPhones transported from O_i to D_j. If we want to formulate the above transportation problem as a mathematical model,

(1) (5 分) We must make this assumption "$\sum_{i=1}^{m} s_i \le \sum_{j=1}^{n} d_j$" for our problem. True or false … (　　).

(2) (5 分) We can use "$\sum_{i=1}^{m} x_{ij} \ge d_j$, for all j" as constraints of our model. True or false … (　　).

(3) (5 分) We can use $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ as part of the objective function (可用來做為目標函數的一部份). True or false … (　　).

(4) (5 分) There is no way (沒有辦法) to formulate this transportation problem as a linear program. True or false … (　　).

注：背面有試題

4. (5 * 5 = 25 points)

Patrons arrive at the NCU post office at the rate of 30 per hour. There is one clerk on duty, who takes an average of 1 minute to serve each customer. Service times are approximately exponential. Calculate

(1) the mean customer time spent waiting in line

(2) the mean customer time spent receiving or waiting for service

(3) the mean number of customers in line

(4) the mean number of customers receiving or waiting for service

(5) the probability there is no customer in line.

5. (5 * 3 = 15 points)

Suppose there are 2 competing products, E and N, in the market. After one year, some customers will keep using the same product but some will switch to using the other product. The proportion of the customers who keep using the same product or switch to use the other product after one year is summarized in the following table:

	E	N
Keep	1/4	1/3
Switch	3/4	2/3

Suppose $\frac{3}{5}$ of the customers use product E and the other $\frac{2}{5}$ of customers use product N in the beginning. Then,

(1) What is the distribution after one year?

(2) What is the distribution after two years?

(3) What is the distribution as the market is said to be stable?

6. (10 points)

Suppose you divide his email into three categories:

• A1 = "spam (垃圾郵件),"

• A2 = "low priority,"

• A3 = "high priority."

From previous experience, you find that

P(A1) = 0.7

P(A2) = 0.2

P(A3) = 0.1 .

Let B be the event that the email contains the word "free." From previous experience,

P(B|A1) = 0.9

P(B|A2) = 0.01

P(B|A3) = 0.01 .

Now, suppose you receive an email with the word "free." What is the probability that it is "spam"?