所別：工業管理研究所碩士班 甲組(一般生)　科目：微積分　共 2 頁 第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

1. (50 points) Consider a manufacturer who produces and sells a certain product to a dedicated retailer who faces uncertain demand in the market. The product is subject to a fixed life-time (such as newspapers, fashion goods, and agricultural produces). Let

c = manufacturing cost per item.
c_1 = price per unit paid by the retailer to the manufacturer.
c_2 = credit per unit paid by the manufacturer to the retailer for returned goods.
c_3 = salvage value per unit.
p = selling price per unit by the retailer.
g = goodwill cost per unit due to stockout incurred by the retailer.
g_1 = additional goodwill cost per unit due to stockout incurred by manufacturer.
$g_2 = g + g_1$ =total goodwill cost.
Q = amount ordered by the retailer from the manufacturer.
R = percentage of order quantity Q which the retailer can return to the manufacturer for a credit of c_2 per item.
$f(x)$ = probability density function of demand; and
$F(k) = \int_0^k f(x)dx$, the cumulative frequency distribution of demand.

The following two relationships on the values are assumed to hold:

$$c_3 < c < c_1 < p$$

and

$$c_3 < c_2 \le c_1 < p.$$

Consider a situation in which the retailer determines the order quantity to purchase from the manufacturer. The retailer's expected profit, *EP(Q)*, will be given by the formula:

$$EP(Q) = -Qc_1 + \int_0^{(1-R)Q} [xp + RQc_2 + ((1-R)Q - x)c_3] f(x)\,dx \qquad EQ(1.1)$$

$$+ \int_{(1-R)Q}^{Q} [xp + (Q-x)c_2] f(x)\,dx \qquad EQ(1.2)$$

$$+ \int_{Q}^{\infty} [pQ - (x-Q)g] f(x)\,dx. \qquad EQ(1.3)$$

(a) (10 points) Explain, in detail, the managerial implications for the third term, i.e., *EQ(1.2)*, of the profit model $EP(Q)$:

$$+ \int_{(1-R)Q}^{Q} [xp + (Q-x)c_2] f(x)\,dx$$

(b) (10 points) Explain, in detail, the managerial implications for the fourth term, i.e., *EQ(1.3)*, of the profit model $EP(Q)$:

$$+ \int_{Q}^{\infty} [pQ - (x-Q)g] f(x)\,dx.$$

(c) (10 points) Find the first order necessary condition (FONC) of $EP(Q)$ (i.e., differentiating $EP(Q)$ with respect to Q and setting this amount equal to zero).

(d) (10 points) Find the optimal purchase quantity Q^* such that the retailer's expected profit is maximized (i.e., solving the FONC).

(e) (10 points) Prove that the second order sufficient condition (SOSC) holds.

參考用

注意：背面有試題

2. A real-valued function f defined in (a,b) is said to be convex if

$$f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda)f(y)$$

, whenever $a < x < b$, $a < y < b$, and $0 < \lambda < 1$.

(a) Show that every convex function is continuous. (10 points)

(b) Show that every increasing convex function of a convex function is convex. (Example, if f is convex, so is e^f.) (10 points)

(c) If f is convex in (a,b) and if $a < s < t < u < b$, show that

$$\frac{f(t)-f(s)}{t-s} \le \frac{f(u)-f(s)}{u-s} \le \frac{f(u)-f(t)}{u-t}$$

(10points)

3. If f is continuous on $[0,1]$ and if

$$\int_0^1 f(x)x^n dx = 0$$

, for $n = 0,1,2,\ldots$ Show that $f(x) = 0$ on $[0,1]$

(Hint: Use the Weierstrass theorem to show that $\int_0^1 f^2(x)dx = 0$) (10 points)

4. Show that very uniformly convergent sequence of bounded function is uniformly bounded. (10 points)

參考用

注意:背面有試題