

國立中央大學97學年度碩士班考試入學試題卷

所別：經濟學系碩士班 科目：統計學 共 3 頁 第 / 頁

\*請在試卷答案卷(卡)內作答

Note: Precise answers to some questions might be unavailable without probability Tables or calculators. In that case, just plug in your numbers in the formula.

1. (16 points) The joint probability distribution of  $X$  and  $Y$  is shown in the accompanying table.

	X		
Y	0	1	2
0	.42	.12	.06
1	.21	.06	.03
2	.07	.02	.01

- Determine the (marginal) probability distributions of  $X$  and  $Y$ .
  - Are  $X$  and  $Y$  independent? Explain.
  - Find  $P(Y = 1|X = 2)$ .
  - Find the probability distribution of  $X + Y$ .
  - Find  $Cov(X, Y)$ .
2. (22 points)  $X$  is a Poisson random variable and its probability distribution is given by

$$Prob(X = x) = f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots$$

where  $\lambda$  is the average number of success occurring in a specifying interval. In addition,  $Y$  is the number of successes in the  $n$  trials of a binomial experiment that has probability  $p$  of a success on any given trial, and its probability distribution is given by

$$Prob(Y = y) = g(y) = C_y^n p^y (1 - p)^{n-y}, \quad y = 0, 1, 2, \dots, n.$$

- Find  $E(X^3)$  and  $E(Y^3)$ .
  - It is known that 3% of the tickets in a certain scratch-and-win game are winners, in the sense that the purchaser of such a ticket will receive a prize. If three tickets are purchased at random, what is the probability that exactly one of the tickets is a winner?
  - The marketing manager of a company has noted that she usually receives 10 complaint calls from customers during a week (consisting of 5 working days) and that the calls occur at random. Find the probability of her receiving exactly 3 such calls in a single day.
3. (6 points) Suppose that the starting monthly salaries of M.B.A.s at West University (WU) are normally distributed with a mean of NT\$32,000 and a standard deviation of NT\$7,000. The starting monthly salaries of M.B.A.s at the East University (EU) are normally distributed with a mean of NT\$30,000 and a standard deviation of NT\$8,000. If a random sample of 60 WU M.B.A.s and a random sample of 45 EU M.B.A.s are selected, what is the probability that the sample mean of WU graduates will exceed that of the EU graduates?

參考用

注意：背面有試題

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所別：經濟學系碩士班 科目：統計學 共 21 頁 第 2 頁

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4. (16 points) ASUS Computers Inc has asked for your assistance in comparing the productivity of their old and new computer models for processing a random sample of commercial applications. A randomly selected set of 25 jobs was selected and the sample mean processing time for the old computer model was 3.6 minutes. The sample mean time for the 25 jobs using the new computer was 2.2 minutes. The population variance for both computers is  $\sigma^2 = 1$  and the correlation between times is 0.5.

- (a) Prepare and carry out a hypothesis test to determine if there is strong evidence that the new computer has a lower mean processing time for the population of jobs. Use a probability of error equal to 0.05.
- (b) Compute the probability of Type II error for selected values of the difference in population means and prepare a power curve.

5. Suppose you used statistical software to fit the model  $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{u}$  to  $n = 15$  data points and the ordinary least square (OLS) results are shown below (18 points)

Predictor	Coefficient	SE Coefficient	T-value	Probability
Constant	90.10	23.10	3.90	0.002
X1	-1.836	0.367	-5.10	0.001

*Analysis of Variance*

Source	DF	SS	MS	F	P
Regression	1	14801	7400	64.91	0.001
Residual error	13	1364	114		
Total	14	16165			

- (a) Interpret the OLS estimator.
- (b) Find  $R^2$  and interpret its value.
- (c) Test the null hypothesis  $H_0: \beta_1 = -1$  against the alternative hypothesis  $H_1: \beta_1 \neq -1$ . Test using  $\alpha = 0.05$ . Draw the appropriate conclusion.
6. To consider the influence of gender on leadership ability, the following  $3 \times 2$  two-way factorial design table gives score on an index that measures leadership ability, for respondents at three levels of management. It is assumed that the populations are normal and independent and the observations are randomly obtained. High scores represent greater leadership ability.

Gender	Supervisor	Middle-Level Manager	Upper-Level Manager
Male	16, 17, 25	18, 25, 30	20, 30, 42
Female	18, 20, 28	20, 28, 30	30, 41, 55

- (a) (10 points) Compute the ANOVA table.
- (b) (6 points) At  $\alpha = 0.05$ , test for a significant difference in leadership among three groups of manager.
- (c) (6 points) Test for interaction between managerial level and gender at  $\alpha = 0.05$ .

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F

$\nu_2$	$\nu_1$	NUMERATOR DEGREES OF FREEDOM								
		1	2	3	4	5	6	7	8	9
1		161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2		18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3		10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4		7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5		6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6		5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7		5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8		5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10		4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11		4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12		4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13		4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14		4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15		4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16		4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17		4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18		4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19		4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21		4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22		4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23		4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24		4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25		4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26		4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27		4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28		4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29		4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40		4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60		4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120		3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
$\infty$		3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

t

DEGREES OF FREEDOM	$t_{\alpha}$					DEGREES OF FREEDOM	$t_{\alpha}$				
	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$		$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657	24	1.318	1.711	2.064	2.492	2.797
2	1.886	2.920	4.303	6.965	9.925	25	1.316	1.708	2.060	2.485	2.787
3	1.638	2.353	3.182	4.541	5.841	26	1.315	1.706	2.056	2.479	2.779
4	1.533	2.132	2.776	3.747	4.604	27	1.314	1.703	2.052	2.473	2.771
5	1.476	2.015	2.571	3.365	4.032	28	1.313	1.701	2.048	2.467	2.763
6	1.440	1.943	2.447	3.143	3.707	29	1.311	1.699	2.045	2.462	2.756
7	1.415	1.895	2.365	2.998	3.499	30	1.310	1.697	2.042	2.457	2.750
8	1.397	1.860	2.306	2.896	3.355	35	1.306	1.690	2.030	2.438	2.724
9	1.383	1.833	2.262	2.821	3.250	40	1.303	1.684	2.021	2.423	2.705
10	1.372	1.812	2.228	2.764	3.169	45	1.301	1.679	2.014	2.412	2.690
11	1.363	1.796	2.201	2.718	3.106	50	1.299	1.676	2.009	2.403	2.678
12	1.356	1.782	2.179	2.681	3.055	60	1.296	1.671	2.000	2.390	2.660
13	1.350	1.771	2.160	2.650	3.012	70	1.294	1.667	1.994	2.381	2.648
14	1.345	1.761	2.145	2.624	2.977	80	1.292	1.664	1.990	2.374	2.639
15	1.341	1.753	2.131	2.602	2.947	90	1.291	1.662	1.987	2.369	2.632
16	1.337	1.746	2.120	2.583	2.921	100	1.290	1.660	1.984	2.364	2.626
17	1.333	1.740	2.110	2.567	2.898	120	1.289	1.658	1.980	2.358	2.617
18	1.330	1.734	2.101	2.552	2.878	140	1.288	1.656	1.977	2.353	2.611
19	1.328	1.729	2.093	2.539	2.861	160	1.287	1.654	1.975	2.350	2.607
20	1.325	1.725	2.086	2.528	2.845	180	1.286	1.653	1.973	2.347	2.603

參考用