

國立中央大學98學年度碩士班考試入學試題卷

所別：經濟學系碩士班 科目：統計學 共二頁 第 / 頁

*請在試卷答案卷(卡)內作答

請在答案紙上按題號順序作答，否則不予計分。

1. The joint pdf for two random variables X and Y is defined by

$$f(x, y) = \begin{cases} C(x+2y) & \text{if } 0 < y < 1 \text{ and } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

- (3分) Find the value of C .
 - (3分) Find the marginal distribution of X .
 - (3分) Compute $E(X)$.
 - (3分) Find the joint cdf of X and Y .
 - (3分) Are X and Y independent to each other? Show how you get your answer.
2. Suppose X_i ($i = 1, \dots, n$) are n independent random variables that are drawn from one identical normal distribution with mean μ and variance σ^2 , namely $X_i \sim \text{iid } N(\mu, \sigma^2)$.
- (5分) Let $Z = 5 - 8X_1 + 6X_2$. Compute the coefficient of correlation between X_1 and Z .
 - (5分) An estimator for the population mean is $\hat{\mu} = \frac{1}{(n+1)} \sum_{i=1}^n X_i$. Suppose $n=9$, $\mu=2$, and $\sigma^2=4$. Compute the mean-square-error (MSE) of $\hat{\mu}$.
 - (5分) Let $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$. Suppose $n=100$, $\mu=0$, and $\sigma^2=4$. Apply the Central Limit Theorem to calculate the probability $\text{Prob}(\bar{X} < -0.392)$.
 - (5分) For $n > m$, what is the probability distribution of $\frac{\sum_{i=1}^m (X_i - \mu)^2 / (m-1)}{\sum_{i=m+1}^n (X_i - \mu)^2 / (n-m-1)}$?
3. Suppose that $\lambda\%$ ($0 < \lambda < 1$) of the population never buy lotteries. Among the remaining $(1-\lambda)\%$ population, the probability of buying lotteries is 0.8 when the lottery stake (彩票獎金) accumulates more than ten million dollars; the probability of buying lotteries is 0.3 when the stake is smaller than ten million dollars. The probability for the lottery to have more than ten million dollars stake is 0.9; the probability for the lottery to have less than ten million dollar stake is 0.1.
- (7分) What is the probability of buying lotteries for a given lottery stake in the whole population? (Remark: the given stake could be larger or smaller than ten million dollars.) (Hint: The answer is a function of λ .)
 - (8分) In a random sample of 100 residents, only 15 of them bought lotteries for a specific lottery stake. Use the Maximum Likelihood Estimation (MLE) method to estimate the unknown parameter λ .

注意：背面有試題

參考用

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4. Answer the following questions.

- (4分) What is the difference between Type I and Type II errors in hypothesis testing? How do α and β relate to Type I and Type II errors?
- (4分) If the average monthly salary of workers with an undergraduate degree of economics is NT\$ 30,000. You collect a sample (large sample, n) and use a 95% confidence interval to estimate the population mean. What is meant by the phrase "95% confidence interval"?
- (4分) You estimate a simple regression and obtain the determinant of coefficient $R^2 = 0.81$. Please interpret the result.
- (4分) Show the formula of Laspeyres' Price Index and indicate its shortcoming.

5. Advertisements by California Fitness Center claim that completing their course will result in losing weight. A random sample of nine participants reported their weights before and after completing the course (See the table below). At the 0.05 significance level, can you conclude that the participants lose weight? (計算至小數點第一位即可)

Name	Before	After	
Andy	155	154	(pounds)
Bernie	228	207	
Chris	141	146	
Denies	162	157	
Eric	211	196	
Fredrick	164	150	
Glashutte	184	170	
Hamilton	172	165	
Peter	120	120	

參考用

- (4分) State the null hypothesis and the alternative hypothesis.
 - (4分) What are the critical value of t and the computed value of t , respectively?
 - (4分) Interpret the result.
 - (4分) What assumption do you need to make about the distribution of the weight differences?
6. A factory manager is interested in modeling $E(y)$, the expected length of time per month (in hours) that a machine will be shut down for repairs, as a function of the type of machine (001 or 002) and the age of the machine (in years). The manager has proposed the following model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2$$

Where x_1 = age of machine

x_2 = 1 if machine type 001, 0 if machine type 002

Using a dataset of $n=20$ to estimate the parameters of this model and obtain

$$\hat{y} = -11.8 + 10.3x_1 - 0.418x_1^2 + 13.2x_2$$

$$Se(3.050) (1.438) (0.161) (1.503)$$

- (5分) Does these data provide sufficient evidence to conclude that the second-order term (x_1^2) in the model proposed by the manager is necessary? Test it using $\alpha=0.05$. (Please specify the hypotheses.)
- (5分) Interpret the estimated coefficient of β_3 .
- (8分) Test the null hypothesis that $\beta_1 = \beta_2 = 0$ using $\alpha=0.05$. Interpret the steps how you carry out this test.

注意：背面有試題