國立中央大學107學年度碩士班考試入學試題

所別： 數學系 碩士班 數學組(一般生) 共1頁 第1頁

科目： 高等微積分

本科考試禁用計算器 *請在答案卷(卡)內作答

參考用

Write legibly and logically. Decide how much details to include.

Part I: Definitions and Theorems. Complete the sentence in the case of a definition

1. (7ps) Let $n \geq 2$, $m \geq 1$ and $f : \mathbb{R}^n \to \mathbb{R}^m$. We say that f is **differentiable** at x_o if ...
2. (7ps) A set $X \subseteq \mathbb{R}^n$ is **open** if ...
3. (7ps) A set $X \subseteq \mathbb{R}^n$ is **sequentially compact** if...
4. (7ps) Let $f, f_k : X \subseteq \mathbb{R}^n \to \mathbb{R}$ where $k = 1, 2, \ldots$. We say that f_k converge **pointwise** to f on X if ....
5. (7ps) Let $f, f_k : X \subseteq \mathbb{R}^n \to \mathbb{R}$ where $k = 1, 2, \ldots$. We say that f_k converge **uniformly** to f on X if ....
6. (7ps) State the Heine-Borel Theorem.
7. (7ps) State the Minimum-Maximum (also known as the Extreme Value) Theorem.

Part II: Computations

8. (10ps) Find $\displaystyle \lim_{x \to 0^+} \frac{\sqrt{ln(sec\, x)}}{x}$.
9. (10ps) Find $\displaystyle \int_0^1 \int_{\sqrt{x}}^1 sin\left(\frac{y^3+1}{2}\right) dy\, dx$.
10. (11ps) Find the maximum and minimum values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint: $x^4 + y^4 + z^4 = 1$.

Part III: Proofs

11. (7ps) Let $f_k : X \subseteq \mathbb{R}^n \to \mathbb{R}$ be a sequence of continuous function. Suppose f_k converge uniformly to f on X. Prove that f is also continuous on X.
12. (3ps) Give an example showing (and write the proof for it) that this is not true if the f_k are just converging pointwise to f.
13. (10ps) True or false? If you think the following statement is false, give a counter-example (and prove that your example works) and if you think that it is true, prove it. Let $X \subset \mathbb{R}^n$ be an open set and $f : X \to \mathbb{R}$, $x_o \in X$, $\vec{n} \in \mathbb{R}^n$ is a unit vector. Suppose f is differentiable in every direction $\vec{n}$ at x_o, then f is differentiable at x_o.