系所： 數學系 碩士班 數學組(一般生)
數學系 碩士班 應用數學組(一般生)
數學系 碩士班 應用數學組(在職生)

科目： 線性代數

*本科考試禁用計算器

Do all problems. Show your work.

1. (10pts). Let $A = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} \in M_{2\times 2}(\mathbb{R})$, $S = \left\{ \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \right\} \subset M_{2\times 2}(\mathbb{R})$.
   (a) Determine whether A is in the span of S.
   (b) Determine whether S is a linearly independent subset of $M_{2\times 2}(\mathbb{R})$.

2. (10pts) Let V be a vector space, and let $T : V \to V$ be a linear transformation. Prove that $T^2 = T_0$ (the zero transformation) if and only if $R(T) \subseteq N(T)$
   [Notations: $N(T)$:= null space of T, $R(T)$:= range of T]

3. (10pts) Let $A, B \in M_{n\times n}(F)$.
   (a) Show that if A is invertible then so is A^t, and $(A^t)^{-1} = (A^{-1})^t$.
   (b) Show that if A and B are similar, then $\det(A - \lambda I) = \det(B - \lambda I)$, $\lambda \in F$.

4. (10ts) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 3 & -1 \end{pmatrix}$. Compute $\det(A)$, the rank of A, and the inverse of A if exists.

注意:背面有試題

國立中央大學114學年度碩士班考試入學試題

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5. (25pts) Let $A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 1 & -4 \\ 2 & 0 & -1 \end{pmatrix} \in M_{3\times 3}(\mathbb{R})$.
   (a) Determine all eigenvalues of A.
   (b) For each eigenvalue λ of A, find the set of eigenvectors corresponding to λ.
   (c) Find the minimal polynomial of A.
   (d) If possible, find a basis for $\mathbb{R}^3$ consisting eigenvectors of A.
   (e) If successful in finding such a basis, determine an invertible matrix Q and a diagonal matrix D so that $Q^{-1}AQ = D$.

6. (15ts) Let $W \subset \mathbb{R}^4$ be the subspace of $\mathbb{R}^4$ generated by the three linearly independent vectors $w_1 = (1,0,1,0)$, $w_2 = (1,1,1,1)$, and $w_3 = (0,1,2,1)$.
   (10pts) (a) Apply Gram-Schmidt process to the set $\{w_1, w_2, w_3\}$ to obtain an orthonormal basis for W.
   (5pts) (b) Find a unit vector $v \in \mathbb{R}^4$ that is in the orthogonal complement $W^\perp \subset \mathbb{R}^4$ of W.

7. (10pts) Let V be an inner product space, and let W be a finite dimensional subspace of V. If $x \notin W$, prove that there exists $y \in V$ such that $y \in W^\perp$ but $\langle x, y\rangle \neq 0$.

8. (10pts) Let T the a linear operator on the inner product space $\mathbb{C}^2$ defined by $T(a,b) = (2a+ib, a+2b)$.
   (a) Determine whether T is normal or not.
   (b) Determine whether T is self-adjoint or not.