參考用

一. Suppose an element a in a group has order n. Prove that $a^t = e$, the identity element, if and only if $t = nq$ for some integer q. (10%)

二. Prove that two cyclic groups of order infinity are isomorphic (10%)

三. Prove that if $|G| = p^2$, p is a prime, then the group G either $G \cong \mathbb{Z}_{p^2}$ or $G \cong \mathbb{Z}_p \times \mathbb{Z}_p$. (20%)

四. A ring A is boolean if $a^2 = a$, $\forall a \in A$. Prove that $a = -a$, $\forall a \in A$; and A is commutative. (10%)

五. Let A be a commutative ring with unity and J an ideal of A. Prove that J is a prime ideal if and only if A/J is an integral domain. (20%)

六. Find all ideal of $\mathbb{R}$, the field of real numbers. (10%)

七. Prove that "Squaring the circle of radius 1" by ruler and compass is impossible. (10%)

八. Find the Galois group of $\mathbb{Q}(\sqrt{2}+\sqrt{3})$ over $\mathbb{Q}$, the field of rational numbers. (10%)