1) 某種賭博遊戲每次贏的機率是19/37，贏了得一元，輸了賠一元。 20%
i)假使要使至少贏1,000元的機率爲1/2，則應玩幾次？
ii)根據 i) 的答案，輸錢的機率是多少？

2) 兩個朋友相約中午在餐廳見面，假設他們從12點到1點隨機抵達餐廳，而且抵達時間互爲獨立。又假設他們用餐時間皆呈均值分佈，從20分鐘到40分鐘，用餐後即離開。他們在餐廳見面的機率是多少？ 10%

3) Determine if the following functions are probability (density) functions. 20%
i) $f(x) = 1 - |1 - x| \qquad 0 < x < 2$
ii) $F(x,y) = \begin{cases} 0 & \text{if } x + y < 1, \\ 1 & \text{if } x + y \geq 1. \end{cases}$

4) Suppose that X is a continuous random variable with density function f. 20%
i) Find M_1, assuming it exists, such that $E(X-a)^2$ is minimized at $a = M_1$.
What is M_1 ?
ii) Find M_2 such that $E|X - a|$ is minimized at $a = M_2$.
What is M_2 ?

5) First determine if the following statements are true. Then prove the statements that are true, or give counterexamples to the false statements. 30 %
Let X and Y be two *independent* random variables. Let A, B and C be three events.
i) $E(\frac{X}{Y}) = \frac{E(X)}{E(Y)}$
ii) X and Y are uncorrelated.
iii) $f(X)$ and $f(Y)$ are independent for any function f.
iv) $P(ABC) = P(A)P(B)P(C)$ implies that the events A, B and C are mutually independent.
v) If events A and B are independent, then they are mutually exclusive.
vi) If A is independent of B and A is independent of C, and $B \cap C = 0$, then A is independent of $B \cup C$.