1. The function $y_1 = \dfrac{\sin x}{\sqrt{x}}$ is a solution of

$$x^2 y'' + x y' + (x^2 - \tfrac{1}{4}) y = 0 \quad \text{on} \quad (0, \pi).$$

   Find a second solution.

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2. Find a general solution of

$$x^3 y''' - 4 x^2 y'' + 8 x y' - 8 y = 4 \ln x.$$

   20%

3. Determine a function $M(x, y)$ so that the following differential equation is exact.

$$M(x, y) \, dx + (x e^{xy} + 2xy + \tfrac{1}{x}) \, dy = 0.$$

   20%

4. Solve

$$y' - 5y = -\tfrac{5}{2} x y^3.$$

   20%

5. Solve

$$y'' + 2y' - 3y = 9x, \qquad y(0) = 1, \quad y'(1) = 2.$$

   20%