# 國立中央大學八十四學年度碩士班研究生入學試題卷

所別：數學研究所　　組　　科目：數值分析　　共 / 頁 第 / 頁

**1.** (10 points) Find the Newton interpolating polynomial for these function values:

| $x$ | 3 | 1 | 5 | 6 |
|---|---|---|---|---|
| $f(x)$ | 1 | −3 | 2 | 4 |

**2.** (20 points) Let

$$A = \begin{pmatrix} 4 & -1 & 1 \\ -1 & 4.25 & 2.75 \\ 1 & 2.75 & 3.5 \end{pmatrix}$$

**2.1** Use Gaussian elimination with scaled row pivoting and determine the factorization of the form $PA = LU$.

**2.2** Find the Cholesky factorizations of the matrix $A$.

**3.1** (10 points) State the Gershogrin's Theorem.

**3.2** (10 points) Without Computing them, prove that the eigenvalues of the matrix

$$A = \begin{pmatrix} -1+i & 0 & \frac{1}{4} \\ \frac{1}{4} & 1 & \frac{1}{4} \\ 1 & 1 & 3 \end{pmatrix}$$

satisfy the inequality $\frac{1}{2} \leq |\lambda| \leq 5$.

**4.** (20 points) Carry out one iteration of Newton's method from $x = 2$ for the function $f(x) = x^4 - 12x^3 + 47x^2 - 60x$. Which root are you converging to? What happens if you start from $x = 1$ ?

**5.** (10 points) Suggest ways to avoid loss of singnificance in these calculations:

**5.1** $x - \sin x$.

**5.2** $\sqrt{x^2+1} - x$.

**6.** (10 points) Suppose that the bisection method is started with the interval $[2, 18]$. How many steps should be taken to compute a root with relative accuracy of one part in $10^{-13}$ ?

**7.** (10 points) Prove that if a matrix $A$ is positive definite and symmetric, then its eigenvalues are real and positive.