1. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find $\det(A)$, $\text{rank}(A)$ and the inverse of A if it exists. (15%)

2. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$ if it exists. (10%)

3. Let $A = \begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix}$ Find a diagonal matrix D and an orthogonal matrix C such that $C^{-1}AC = D$. (15%)

4. Let $A = \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix}$. Find A^{15}. (15%)

5. Let T be a linear transformation from the vector space V to the vector space W. Let X be a finite dimensional subspace of V and Y is the image $T(X)$ of X. Show that Y is a subspace of W and $\dim(Y) \leq \dim(X)$. (15%)

6. Let $W = \{(x_1, x_2, x_3, x_4) : x_1 + x_2 - x_3 - x_4 = 0\}$. Let T be the projection of $\mathbf{R}^4$ on W. Represent T as a matrix with respect to the standard basis. (15%)

7. Let T be a linear transformation from a finite dimensional vector space V to itself. Then there is a nonzero polynomial $p(x)$ such that $p(T)$ is a zero transformation.(15%)